Geometry of torus bundles in integrable Hamiltonian systems

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Monodromy in non-integrable systems on certain compact classical phase spaces. Physics Letters A, Vol. 383, Issue. 5, p. 452.Â We obtain a global version of the Hamiltonian KAM theorem for invariant Lagrangian tori by gluing together local KAM conjugacies with the help of a partition of unity. In this way we find a global Whitney smooth conjugacy between a nearly integrable system and an integrable one. This leads to the preservation of geometry, which allows us to define all non-trivial geometric invariants of an integrable Hamiltonian system (like monodromy) for a nearly integrable one. Export citation Request permission. Copyright. We obtain a global version of the Hamiltonian KAM theorem for invariant Lagrangean tori by glueing together local KAM conjugacies with help of a partition of...Â In this way we find a global Whitney smooth conjugacy between a nearly-integrable system and an integrable one. This leads to preservation of geometry, which allows us to define all the nontrivial geometric invariants like monodromy or Chern classes of an integrable system also for near integrable systems. External-identifier. urn:arXiv:math/0210043. Mathematics > Dynamical Systems. Title:Geometry of integrable non-Hamiltonian systems. Authors:Nguyen Tien Zung. (Submitted on 16 Jul 2014). Abstract: This is an expanded version of the lecture notes for a minicourse that I gave at a summer school called "Advanced Course on Geometry and Dynamics of Integrable Systems" at CRM Barcelona, 9--14/September/2013. In this text we study the following aspects of integrable non-Hamiltonian systems: local and semi-local normal forms and associated torus actions for integrable systems, and the geometry of integrable systems of type $(n,0)$. M