The Essential Guide to Secondary Mathematics

Successful and enjoyable teaching and learning

Colin Foster

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Introduction

This book is not just for those who are beginning as mathematics teachers; it is intended just as much for those with more experience. It deliberately avoids dwelling on the intricacies of the particular requirements for passing an initial teacher education course or acquiring newly-qualified-teacher status. Instead, it aims to be about mathematics and the learning of mathematics as ends in themselves. Each chapter is punctuated at intervals with ***Tasks***, some purely mathematical and others more reflective. Some of the mathematical tasks could be used in secondary mathematics classrooms, but they are primarily intended for the reader to engage with. By working regularly on mathematics as you go through the book, you will find that many of the issues discussed will be brought into sharper focus. The reflective tasks are intended to give you the opportunity to ponder what you think about a particular issue. There are rarely simple answers and you will certainly take a different view from me on some of the issues. Some of the tasks are intended to be provocative and destabilizing; some you may find unhelpful—in which case, please just move on to whatever comes next.

No book can tell anyone how to teach mathematics any more than any colleague or mentor can. Every mathematics teacher must find their own way to bring to life the beliefs that they hold about mathematics, about education and about human beings. So this book doesn't attempt to force a particular viewpoint on you, although neither does it seek to conceal my own views. Different perspectives are offered and questions asked, allowing you space to find your own position. I assume that every mathematics teacher, no matter how long in the tooth, can find ways in which they might develop their practice in the classroom. It is a truism to say that to be a teacher is to be a learner too. This book is a tool for mathematics teachers who wish to change and become better at the job that they do. I hope that beginning teachers will return to it at later stages in their career and that it will be of interest to teachers with experience in another subject area (or a different age range) who will be taking on some secondary mathematics classes and might find some differences from what they are used to doing.

Pressures on the mathematics teacher are increasing all the time, with ever more detailed and prescriptive 'guidance'/'support'/interference from those who may not always seem to understand the reality 'on the ground'. It is important for mathematics teachers to have the knowledge and confidence to work with young learners of mathematics in ways that they judge to be appropriate for their situation. All mathematics teachers should be well informed about different viewpoints and possible ways of working, but no one else can know your class as you do. As mathematics teachers, we have

the opportunity and responsibility to offer mathematical tasks that will promote our learners' development in, and love for, mathematics, and to support them as they work on such tasks. This book aims to help us in that important work.

Colin Foster
January 2012

Chapter 1

The mathematics teacher

> I don't understand why anyone would want to be a maths teacher!
>
> Parent at parents' evening

Since this is a book about mathematics teaching, let's begin with some mathematics.

Task Paint

A painter mixes 5 litres of white paint with 3 litres of blue paint.
Then they realize that they meant to do it the other way round.
What is the least wasteful way now of getting the desired colour in sufficient quantity?

Did you try this task? Throughout this book, there will be different kinds of tasks, some mathematical and some not. Perhaps you have seen it (or a clone of it) before, or used it with learners, maybe recently. The tasks in this book will inevitably be 'hit and miss' to some extent; as the reader, you should *use* this book rather than *submit* to it. If you don't find something useful, move on to the next part. Sometimes I find that I am in the mood for working on mathematics; other times I am not. Some tasks immediately grab me; others don't. Of course, such variations in motivation will be true of the learners we work with as well as of ourselves (Middleton and Spanias, 1999). You will find comments on the mathematical tasks at the end of each chapter.

1.1 Motivation

> Mathematics is the queen of the sciences.
>
> Carl Friedrich Gauss

Here is a different kind of task.

Task Why teach mathematics?

Why are you reading this book?
What is your interest in mathematics teaching?
How did you get to this point?

Mathematics teachers come from a variety of backgrounds (Richardson and Watt, 2005). You may be completely new to teaching, or you might be moving to mathematics teaching from teaching something else. Or you might have lots of experience as a mathematics teacher but want to re-think what you do and come at your work in a fresh way. Some mathematics teachers have had a previous career, either in a mathematics-related area or in something else. Some mathematics teachers have mathematics degrees, some have degrees in closely related subjects and some don't. Whatever your particular background, experiences and interests, these will be your strengths and will help to inform the kind of mathematics teacher that you become. It is sensible to view your past positively, since you can't change it, and to draw on whatever advantages it may give you.

People have all sorts of reasons for becoming mathematics teachers (Hutchings, 1996). Sometimes it runs in families (Watt *et al.*, 2007), and even though someone's parents may well have advised them against going into the teaching profession, they did nonetheless. Well-qualified mathematicians are much in demand in a range of occupations, so mathematics teaching is often a conscious choice to accept a significantly lower salary than might be available elsewhere. For some, the termly rhythm and daily hours of the teaching life fit in well with family commitments. For whatever reasons, teaching exerts a powerful draw on many people, often from a young age, at the same time as being widely recognized as a demanding and difficult job.

Task Alternatives

I admire people who teach maths, but I couldn't do it myself. I couldn't stand teaching the same things over and over again.

Research mathematician

What do you think you would be if you *weren't* a mathematics teacher?

Some mathematics teachers cannot imagine being anything else: they don't know what they would do in a world in which mathematics teachers were not required! On the other hand, if you have had a previous career, then this could be your answer. Or perhaps you have a future aspiration and mathematics teaching is something you are doing in between other things. If you weren't a mathematics teacher, maybe you would do something completely unrelated to mathematics or teaching. However, many mathematics teachers, when asked this question, talk either about something mathematical or something educational.

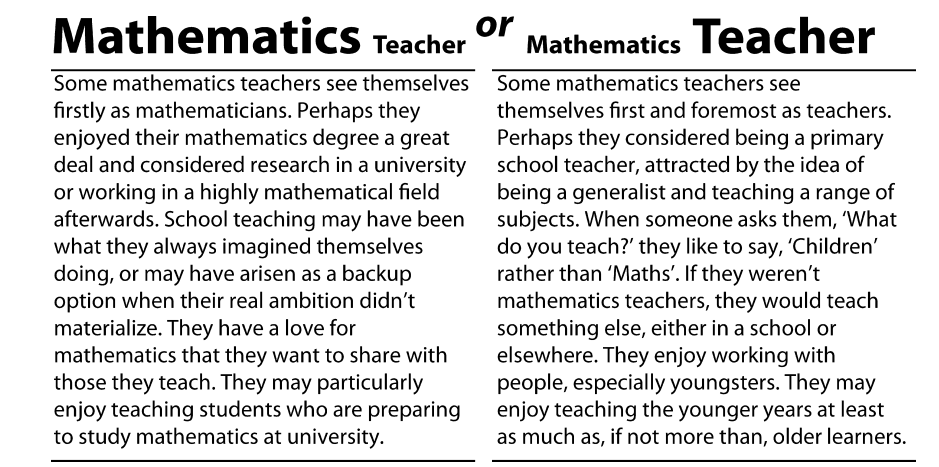


Figure 1.1 Some stereotypes of mathematics teachers

Figure 1.1 characterizes mathematics teachers as firstly teachers or as firstly mathematicians. Of course, these portraits are stereotypes, and many mathematics teachers will position themselves somewhere between these extremes, but these descriptions may help you to locate your chief motivations in being a mathematics teacher. Understanding this may help you to grow into the sort of mathematics teacher that reflects who you really are.

Task **Stereotypes**

You can tell an extrovert mathematician because when you're talking to them they look at your *toes rather than* theirs.

Old joke

What different difficulties might each of the two stereotypical mathematics teachers portrayed in Figure 1.1 experience in school?
Do you find yourself identifying more closely with one than with the other?

For evidence that mathematicians have something of an image problem in Western society today, you need look no further than popular books and films. Rensaa (2006: 2) comments that 'a widespread public image of mathematics is that it is difficult, cold, abstract and in many cultures, largely masculine'. Mendick (2002: 44) concludes that 'The dominant discourse around mathematicians in popular culture depicts them as boring, obsessed with the irrelevant, socially incompetent, male, and unsuccessfully heterosexual'. A study in which children were asked to 'draw a mathematician at work' predictably led to a lot of badly-dressed bald white men in glasses (Picker and Berry, 2000). (Although such depictions were highly stereotypical, many of their drawings do look rather like people I know!)

Such a character might be expected to be poorly adapted for life in a classroom, and many beginning mathematics teachers do experience something of a culture shock if they move directly from being an undergraduate mathematics student to teaching ratio to a class of 11-year-olds. Those with a more introverted personality can initially find it difficult to establish a comfortable classroom manner, yet in the end frequently make popular and effective teachers. To be a successful teacher, it is not necessary to try to be like the most outgoing person you know – great teachers come in all types and it is healthy for learners to work with a wide variety of different kinds of adults in school. Learners' prejudices can become self-perpetuating if only certain types of children think that working with mathematics could be for them. Ideally, within a mathematics department a variety of different types of teachers, male and female, from different backgrounds and with different personalities, will provide role models for many different learners.

At the other end of the spectrum, some teachers seem comfortable in the classroom from the very beginning but lack confidence over their mathematical knowledge. Sometimes teachers are just rusty if they haven't used some of their mathematics for some years, and they will find it coming back as they go on. Others may need to address their subject knowledge more explicitly, particularly if they are teaching older learners for the first time. Even mathematicians who have been successful in obtaining a university mathematics degree sometimes later say that they don't feel that they really understood many basic things thoroughly until they began teaching them and dealing with learners' questions and ideas. Preparing to teach something can be an effective way to learn (or re-learn) it. If you know that you have mathematical weaknesses that could harm your teaching, it is best to be honest about it and begin to address them, perhaps with support from a knowledgeable colleague.

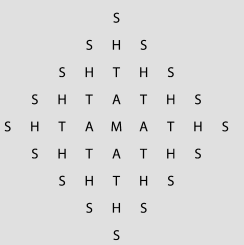
1.2 Mathematics as a subject

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty.
>
> Bertrand Russell

It is one thing to enjoy mathematics yourself, as an interest to read books about it, solve puzzles, talk about it, watch television documentaries about it, or use it in your life or work. It is quite another thing to *impose* mathematics on other people who may not feel the same way about it or think that they need it or can do it. Do mathematics teachers have the right to do that? Mathematics teachers who feel uneasy about this can find it difficult in the classroom when learners do not respond as enthusiastically as they might wish.

Task

Maze



Moving from letter to letter, in how many ways can you spell MATHS?
Are you sure that you have found all the possible ways?
Can you be certain that you didn't miss any? How?
What happens with other words?

Did you do this task? If you did, did you find it fun? Did you think to try *palindromic* words as well? Do you think that you learned anything from doing it? Is it the sort of task that you would use with learners? For some people, puzzles such as this are enjoyable and interesting in their own right: there is a self-imposed challenge in using your powers to solve something, which many people relish. A lot of mathematics teachers enjoy crosswords or Sudoku puzzles, but no one thinks that these things have worldly importance. Unless it is for a competition, once you complete the puzzle, it probably goes straight in the bin – the final answer is of no importance; only the experience of the doing. But not all learners will be excited about working on puzzles.

Task Why learn mathematics?

Why do we have to learn this? When will we ever actually use this?
Learners of mathematics

Why do you think mathematics is worth learning?
Do you believe in compulsory mathematics education? If so, up to what age? Why?

It can be very helpful to think about what you believe about these issues before you go into the mathematics classroom (Noyes, 2007). Of course, there are no right or wrong answers, but knowing what you think can give you the confidence to engage with some of the sorts of things that learners may say (Ernest, 2000; Heymann, 2003). For some, mathematics possesses intrinsic interest, doing mathematics is fun and no further justification is needed. Many mathematics teachers may feel that way, but inevitably when you are teaching a compulsory subject not all learners will share your perspective. You might hope to change learners' minds about this over a period of time, but in the meantime it may be more realistic to accept that some learners will, at best, see mathematics as a means to an end (a service subject) and, at worst, as an unwarranted intrusion into their lives! Even some who do well in mathematics (as measured by test results) may harbour negative perceptions of the subject. Noddings (2003: 202) comments that 'Huge numbers of ... "successful" students finish their schooling with a fear and loathing of mathematics that will last a lifetime'.

Task Responses

You might like to think about the following responses that a mathematics teacher might make and the assumptions that lie behind each.
What are the possible benefits of responding in each of these ways? What are the dangers?

- Maths is useful in all sorts of everyday situations, such as home DIY or managing your money. If you don't understand maths, you'll get taken advantage of in

life – you won't be able to check your payslip or get a good financial deal; you'll be conned whenever you buy anything!
- The world is getting more and more technological and more and more people need more and more maths in all sorts of jobs these days. Employers consistently say that they want numerate people.
- OK, you may not use this particular bit of maths in the future, but it's all good exercise for your brain, making you think, strengthening all those brain cells in there! And you're going to use your brain in the future, whatever you end up doing!
- You don't come to school mainly to get vocational training for your future career. This is your *general* education – it makes you more human to learn about a range of things, many of which you probably won't be doing yourself in the future. It's about understanding the world around you and finding out what you like and what other people do.
- Because it's fun! Even if you're not enjoying this, stick with it and I think that by the end of term you'll be liking maths at least a little bit more than you are at the moment.

1.2.1 Utility for life

Arguments based on the usefulness of mathematics in everyday life depend on what someone else envisages as 'everyday life'. Many learners don't live in houses that their families own, so the opportunities (or money) for home DIY may be considerably more limited than they might be for the typically middle-class mathematics teacher. At the other extreme, learners from wealthier backgrounds might not readily envisage themselves buying and sawing up wood: they would get a carpenter to do it. Another problem is that the mathematics of everyday life tends to focus on money, length/area/volume calculations and probability. So it is harder to use this sort of argument if your lesson is on a pure mathematics topic such as the irrationality of $\sqrt{2}$. Some mathematics educators have argued that real mathematics is not actually very useful in ordinary life (Andrews, 1998), while others have strongly disagreed (Huckstep, 2003). Noddings (2003), for instance, believes that algebra and geometry as currently offered in schools are much less vital to the majority of learners (or for society) than, say, learning about parenting and interpersonal relations. Spencer (1910: 30) suggests that an archaeologist from the future examining our school books and examination papers would conclude that this must have been a curriculum for people who were not going to have children, since there was no mention of them.

1.2.2 Utility for careers

Again, if the learner replies that they are going to be a pop singer, it may be hard to convince them why mathematics (beyond counting their money) is likely to be essential to them. Many learners may anticipate lengthy periods of unemployment and are unlikely to be swayed by possible benefits that might seem to be a long way off. Learners who

'know' that they are going to be an accountant or an engineer, for instance, might be more inclined to appreciate the importance of the subject for their future. However, that does not necessarily mean that they will enjoy today's lesson any more – they might just be more willing to 'suffer' it, which is hardly ideal. There are serious problems with an over-emphasis on the economic benefits of education; for example, Noddings (2003: 4) remarks that 'It is as though our society has simply decided that the purpose of schooling is economic – to improve the financial condition of individuals and to advance the prosperity of the nation'. For many mathematics teachers, this is not what it is all about.

1.2.3 Brain training

Mathematics may be one way of 'exercising' the brain, but it is by no means the only one; learners may prefer to exercise other parts of the brain in other ways. Also, like the previous two responses, this seems to regard mathematics as merely a means to an end, possibly devaluing it as a subject in its own right. After all, memorizing the telephone directory would be a mental challenge, but the benefit would hardly be worth the effort. There must be more to learning mathematics than getting a mental workout.

1.2.4 Learning for a better society

If the learner has a very different philosophy of life, this response may fall on deaf ears, but it is hard to argue with the view that education should be enriching and empowering. It is only by tasting a wide range of disciplines that learners can discover where their particular interests may lie. Understanding what people very different from ourselves are doing in their lives helps to make a cohesive democratic society (Povey, 2003), and, according to Russell (2009: 113–4), 'The defence of the state in all civilised countries is quite as much in the hands of teachers as in those of the armed forces. ... Teachers are more than any other class the guardians of civilisation'.

1.2.5 Enjoyment

It is hard to argue against pleasure! Not everyone enjoys mathematics but many learners do, at least some of the time, or can do if given more positive experiences of it. Even if they are not spectacularly successful in traditional examinations, many learners obtain a lot of satisfaction from solving problems and getting definite solutions. The coherence and power of mathematics appeals to many learners, even if they may not always wish to admit it in front of their peers. Some learners begin to enjoy a subject only when they have reached a certain stage in it, so it may be necessary to persevere with some things for a little while before the enjoyment comes. Subsequent chapters will suggest ways of promoting enjoyment in the learning of mathematics.

Of course, there may be an agenda behind the question. It is wise to take learners' questions seriously but to be aware that a question like this can sometimes be equivalent to a statement, and that the learner may not really want an answer. Such a question may reveal a healthy scepticism regarding the education process; during their teenage years, many learners experience a great desire to leave school and make their own way in life, and questioning the curriculum and those who run it may be part of that. Sometimes a

particularly difficult or unstimulating mathematical task may provoke learners into asking 'why' out of frustration, or it may arise out of an extended period of disillusionment with the subject. Acknowledging these feelings may be more helpful than giving clever answers.

Task Responding

How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?

Albert Einstein

Can you think of other types of response than the five given?
Which sort of response resonates most strongly with you? Why do you think that is?
What sort of approach would/do you use in the classroom?

Pre-prepared answers that are trotted out thoughtlessly are unlikely to be helpful, and getting to know your learners so that you can relate what you say to their perspectives and interests is vital. You may not be able to convince everyone that mathematics is as important as you think it is, and it may be wise to see their view of mathematics as a 'work in progress'.

1.3 Your perspective

I love it when something clicks mathematically for a pupil and it's as if a light has come on.

Mathematics teacher

The discussion above presupposes that 'mathematics' as taught in schools corresponds to 'mathematics' as otherwise understood, but that is not necessarily the case. There has always been lively discussion over what should go into the school mathematics curriculum (Robitaille and Dirks, 1982; Noss, 1994). There are many competing interests, from the academic demands of universities (both for mathematics courses and for courses that *use* some mathematics) to the wide-ranging perspectives of industry, to those who think that education should be an end in itself, not something that panders to changing economic and technological requirements. The debate sometimes polarizes into traditionalist advocates of *back to basics* on the one hand, with an emphasis on mental numeracy and old-fashioned pencil-and-paper calculations, and *progressives* on the other, who seek to draw on more modern developments in mathematics that are seen as relating more closely to the needs of today's society, including appropriate utilization of modern technology (O'Brien, 2007). The *new maths* movement had its heyday in the 1960s, but has received criticism (Kline, 1976). There are equally heated debates over mathematics *pedagogy* (how the subject is taught), and many of these issues will be encountered in later chapters.

Task Discovered or invented?

Mathematics is the language with which God has written the universe.
Galileo Galilei

Do you think that mathematics is discovered or invented? Why?
Can we know that mathematics is true?

You may not find it easy to decide, but your views on this are likely to influence how you teach, and it is worth considering what you believe (Mazur, 2008; Beswick, 2007; Sarukkai, 2005; Hamming, 1980). It can also be an interesting issue to raise with learners (Rowlands and Davies, 2006), for instance by asking what they think mathematics could be like in another part of the universe (Hamming, 1998). Many mathematics teachers seem to take an *absolutist* perspective on the subject, believing that mathematical claims, once proved, are perfectly and completely true forever, independent of the opinions of human beings (Rowlands *et al.*, 2001). The opposite *fallibilist* view (Ernest, 1999) stresses that mathematics is a human enterprise, where mistakes can be made from time to time, so ultimately all results are open to revision. The proof of some theorems requires extremely detailed checking by numerous mathematicians (Singh, 2002) or the verification of thousands of special cases, which is sometimes feasible only by computer (Wilson, 2002). In cases such as these, how many people can really claim to 'know' for themselves that those theorems are true?

It is often assumed that mathematics teachers who take an absolutist perspective will tend to see the subject as an unchanging body of knowledge which must be 'passed on' to their learners, leading to dull chalk-and-talk *delivery* teaching approaches and an absence of creative thinking (Povey, 2002), although this link has been challenged (Rowlands *et al.*, 2011). Certainly, there are as many perspectives on mathematics teaching as there are mathematics teachers, as the following task may reveal.

Task Metaphors

It's like keeeping a whole load of plates spinning constantly.
Mathematics teacher

What other metaphors for mathematics teaching can you think of?
Are they for teaching generally or more specific to mathematics?
Which metaphors do you identify most closely with? Why do you think that might be?

When considering how to view the practice of teaching, stark contrasts are sometimes offered, such as 'a sage on the stage or a guide on the side' (King, 1993). Either you are going to stand at the front of the classroom and broadcast your wisdom, while learners listen meekly and take notes, or you are going to sit in the passenger seat and watch

passively while the learners take complete control of every decision. More commonly, some middle ground is found between these extremes.

Some people think of teaching primarily as one of the *caring* professions, like social work or counselling. Others think medically, drawing parallels with a doctor or psychiatrist, where diagnoses of difficulties are made and treatments prescribed. Some might focus primarily on management issues ('crowd control') and think of police officers, bouncers, prison officers, babysitters or even zoo keepers! Others might think of a life coach or relate teaching mathematics to working on mathematical problems in academia or industry. Metaphors emphasize one aspect to the detriment of all others, and this is their advantage as well as their drawback. The language that you use in relation to teaching mathematics can become habitual and affect the direction that your teaching takes, so it is sensible from time to time to review whether the way you talk about your work supports what you want to achieve or frustrates it. Some of the subsequent chapters will take up some of these ideas.

Task Joys

Teaching maths can be difficult, but there are always surprises to keep you on your toes!

Mathematics teacher

What are some of the joys of teaching mathematics that are most important for you?
(You might need to anticipate if you have not done much mathematics teaching yet.)

Many teachers report high levels of satisfaction with their profession (Chapman and Lowther, 1982). The enthusiasm of young people and their wide-eyed interest in the world around them can be very touching. Seeing 'lights come on' as a learner understands a mathematical idea for the first time is extremely rewarding, as is watching as learners take control of some mathematics, perhaps making conjectures or proving something for themselves. Being involved at such an important stage in other human beings' lives and being present when important ideas are taking root can feel like a privilege. There is much pleasure in seeing young people mature – in many social ways, but also mathematically, progressing and growing in confidence. Being paid to work on mathematics is enough for some mathematics teachers, and of course some teachers just enjoy having a captive audience (at least some of the time)!

For most mathematics teachers, teaching isn't just something they do during the day and then forget about; like any profession which involves working with people, it can become an identity and affect how you relate to people outside work. When off duty, mathematics teachers can find themselves thinking from an educational perspective about children (including their own) and looking for mathematics wherever they go. Many teachers find it hard to switch off during term time, and if you begin to feel trapped in your work–life (im-)balance, experiencing insomnia or anxiety, it is definitely time to

seek help and not to accept this as normal (see chapter 3). All of your experiences will affect how you fill out your role as a teacher of mathematics. As a teacher, you bring to the classroom the totality of your life up to that point: your experiences, your attitudes, your beliefs. Nobody's background is intrinsically better than anyone else's, but it can be useful to think about how your 'mathematics teacher' side can benefit from everything else that is you.

The *Association of Teachers of Mathematics* (www.atm.org.uk) has four 'Guiding Principles', which can be a very good starting point for thinking about what it means to be involved in mathematics teaching. They are reproduced below for your consideration.

- The ability to operate mathematically is an aspect of human functioning which is as universal as language itself. Attention needs constantly to be drawn to this fact. Any possibility of intimidating with mathematical expertise is to be avoided.
- The power to learn rests with the learner. Teaching has a subordinate role. The teacher has a duty to seek out ways to engage the power of the learner.
- It is important to examine critically approaches to teaching and to explore new possibilities, whether deriving from research, from technological developments or from the imaginative and insightful ideas of others.
- Teaching and learning are cooperative activities. Encouraging a questioning approach and giving due attention to the ideas of others are attitudes to be encouraged. Influence is best sought by building networks of contacts in professional circles. (taken from www.atm.org.uk/about/)

Task **Summary task**

People who become teachers haven't really grown up: they want to spend their whole lives going to school.
At least you get long holidays!
Those who can, do; those who can't, teach.

Parents

What motivates you to teach mathematics?
Write down some of the characteristics that you aspire to as a mathematics teacher.
Are there any things that you particularly want to *avoid*?

Comments on mathematical tasks

Task: Paint

Children in the twenty-first century are very used to clicking 'undo' and can be quite flummoxed by situations such as this, in which that is just not possible! There are many real-life modelling issues to contend with (see pages 47–51). Perhaps the painter could

just live with the paler colour? Perhaps they cannot afford to buy more paint, or it is out of stock? But in the spirit of puzzles such as this, they will need to pour off $\frac{2}{5}$ of the mixture, to obtain a mixture containing 3 litres of white and 1.8 litres of blue. Then they need to supply another 3.2 litres of blue paint to make 8 litres of the right colour.

It is interesting to generalize to a starting point of w litres of white and b litres of blue and also to consider what happens if blue paint is more expensive than white (e.g., suppose it costs twice as much) – does it make a difference if we seek the *cheapest* solution rather than the one that is least wasteful of paint? What if white paint is more hazardous to dispose of?

Task: Maze

If you know about *Pascal's triangle*, that might help you. It is interesting to explore words of different lengths (maybe start with MA or MAT and work upwards – to MATHEMATICS, perhaps). Is it just the number of letters in the word that matters? A palindromic word such as LEVEL requires more thought. You might also try extending the idea to three dimensions.

Further reading

Holt, J. (1990) *How Children Fail*, London: Penguin.
Ollerton, M. (2009) *The Mathematics Teacher's Handbook*, London: Continuum.

Chapter 2

Developing as a mathematics teacher

> They say that the first 40 years are the most difficult; after that, it's fine.
>
> Mathematics teacher

You may be an experienced mathematics teacher already, or you may be just starting out. Whatever point you are at, developing in your role is important to any professional. Most teachers would say that they want to be a better teacher in a year's time than they are now. In this chapter, we will look at some ways to develop your practice as a mathematics teacher, and many of these themes will be developed further throughout the rest of the book.

2.1 Learning for yourself

> Everyone has been to school, so every person you meet thinks they automatically know how to teach.
>
> Mathematics teacher

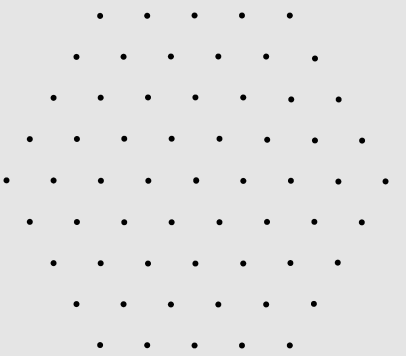
Everyone brings experiences that are relevant to being a teacher of mathematics, the most important of which is that of being a learner of mathematics. No doctor can have personally suffered all of the illnesses that they will treat during their career, and no mathematics teacher will have experienced every possible difficulty with every possible part of the subject. However, every mathematics teacher will have learned for themselves everything that they will teach. You cannot be a mathematics teacher without also being a learner in at least three ways: a learner of mathematics, a learner of mathematics pedagogy and a learner of the learners that you teach.

2.1.1 A learner of mathematics

> A teacher who is not always thinking about solving problems – ones he does not know the answer to – is psychologically simply not prepared to teach problem solving to his students.
>
> Paul Halmos

With this in mind, here is another mathematical task to try (with comments, as usual, at the end of the chapter).

Task Dots



How many dots are there in this drawing?
Can you find out without counting them all?
Make another dotty picture and try to do the same.

Not surprisingly, if you are going to teach mathematics it is useful if you know something about the subject yourself (McNamara, 1991)! Teachers' mathematical knowledge (Shulman, 1986) cannot always be taken for granted (Rowland *et al.*, 2005); indeed, it may be that there is currently a crisis of *subject content knowledge* among mathematics teachers, and a particular shortage of mathematics specialists in primary schools (Goulding *et al.*, 2002). Not all secondary mathematics teachers are confident to teach all classes or all sixth-form options.

However, as well as having learned mathematics in the past, it is important that mathematics teachers continue to see themselves as mathematics *learners* throughout their working lives (and many will find time for even more recreational mathematics in retirement). Working with learners of mathematics on a daily basis can be an effective way of developing your subject knowledge, provided that you take advantage of the opportunities that arise. Learners have a habit of asking simple-sounding but deep questions about mathematics. It is common even for mathematics teachers who have strong mathematical backgrounds to be fazed sometimes by a mathematical question from a young learner.

Task Learning

I have never let my schooling interfere with my education.

Mark Twain

When did you last learn something mathematical?
What was it?
Was it planned or unplanned?
Were you alone? Did someone 'teach' you?

Any mathematics teacher will have learned their mathematics in a variety of ways, ranging perhaps from the traditional university degree to various courses and conferences, private reading and informal conversations. It is much too simplistic to judge someone's knowledge of mathematics merely on the basis of their formal qualifications: biographies of professional mathematicians reveal that many have had quite unconventional mathematical backgrounds (for example, see Kaluza, 2005). For suggestions on ways of developing your mathematical knowledge, see chapter 3.

Task Mathematical knowledge

When I taught Further Maths, I just kept a lesson or two ahead of the class, and it worked fine.

Mathematics teacher

How much mathematics do you think a teacher needs to know to teach pupils up to age 11, age 16 and age 18?

How important do you think that a mathematics degree is for a mathematics teacher?

Teachers are sometimes guilty of assuming that their particular background is best and are less accepting of the routes taken by their colleagues. It is helpful to see every mathematics teacher's subject knowledge as a 'work in progress' and to recognize that someone who knows less content might nonetheless have highly mathematical *habits* that enable them to solve problems unlike any that they have ever seen before or specifically learned to tackle. There are many ways to be a good mathematician and it is restrictive to apply inflexible criteria to all mathematics teachers.

There is no denying that subject knowledge does matter. Knowing things beyond the learners' *mathematical horizon* can be enriching in the classroom (Zazkis and Mamolo, 2011). For example, it may be helpful, when teaching that the angles in a triangle add up to 180°, to know a little about *spherical triangles*. It may help when teaching about corresponding angles to know about the *parallel postulate* and *elliptical* and *hyperbolic geometry* – not necessarily in detail, but knowing that they exist brings out the distinctiveness of Euclidean geometry more starkly (Mlodinow, 2001). It may be important when teaching equivalent fractions to know that not all decimals can be represented exactly as a fraction. If a learner asks why their calculator gives an error when they try to work out the square root of a negative number, it may be helpful to know about *imaginary numbers*. If you are teaching the *scalar product* to sixth formers, it may be helpful to know that there is also a *vector product*, which explains why it is preferable to use a dot rather than a cross for it. There is evidence that weaker subject knowledge tends to result in teachers being more *procedural* in the classroom, teaching learners to memorize rules rather than teaching for understanding (Goulding *et al.*, 2002).

Task Questions

How might you react to these questions from learners?

Why isn't pi exactly 3?
If you zoom in on a circle, is it really a straight line?
Do parallel lines meet at infinity?
Is infinity an actual number?

It is certainly possible to help someone with their mathematics even when you yourself do not know 'the answer'. This is normal behaviour for research mathematicians at coffee time. No one can know all mathematics or the answer to every question a learner (of any age) might ask. Mathematics is a rapidly changing field and it can be hard to keep up-to-date with which conjectures have become proofs and with different applications of the subject. When faced with your own ignorance, as we all are from time to time, it is good to make the most of this opportunity to recognize what it feels like *not* to know, as this can improve your ability to empathize with your learners. It is more important to *be mathematical* than to be a walking encyclopaedia; frequently, an answer to a specific question can be found on the internet (e.g., at www.wolframalpha.com). More often, the situation is well served by asking mathematical questions and *thinking out loud*. Many mathematics teachers will identify with the situation where they are standing at the board working on a problem and cannot immediately see what to do next. Or perhaps you realize that you must have made a mistake but cannot straightaway pinpoint it exactly. The uncertainty and confusion may last for just a few seconds (though it can feel like an eternity!) or it may take much longer to resolve.

Of course, if you have just 'proved' that $2 = 1$, or otherwise realize that you have made a mistake, it is best to admit it without shame as one of those things that happens to everyone from time to time. If you act as though you have done something terrible, learners may mirror this response when they are stuck. In such situations, you have several options:

- *Talk about what you are thinking.*
  For example, 'OK, I can see I've gone wrong here because that can't be that because… So I'm looking back and I know this line is right because … So I'm thinking maybe I made a mistake here because …' Learners may find this more enlightening than the presentation of a polished solution. Such occasions are good opportunities to model *error-correction techniques* and how to deal with being stuck. Sometimes a line of algebra can be checked by trying a particular value, or using dimensions or a diagram can help. Sometimes reversing a process enables you to see what doesn't match up, or trying a different method or simplifying the problem. There will be more about *being mathematical* in chapter 3.
- *Sit down and let a learner take over.*
  This can work well if learners have already solved the problem that you are discussing—sometimes you may feel that everyone in the room except you knows how to do it!

- *Ask learners in pairs or groups to try to resolve the difficulty.*
  With the spotlight off you for a moment, it often takes only a short time to get to grips with it.
- *Return to it next lesson after having a think.*

Not every mathematics teacher is comfortable with being watched when they are stuck. Struggling in public can be hard, and some will be tempted to rub everything out straightaway and say that they will look at it later and discuss it in the next lesson. However, if you can handle it, the experience of *shared stuckness* can be very beneficial. Sometimes when learners describe how their mathematics teacher solves problems on the board they use words like 'magic'. Yet if you want to learn magic there can be an advantage in watching less good magicians, who make mistakes or do things slightly clumsily, rather than those with a faultless technique. The slightly less polished ones enable you to see what they are doing more easily, and perhaps you are slightly less in awe and more open to imagining doing the same things yourself. Of course, if you are regularly making numerous errors in the classroom, and being helped by the learners, then that is likely to become a barrier to their learning, and it will be important for you to seek support and work on boosting your subject knowledge.

Task Integration

For which values of p and q can you find the following integrals?

$$\int x^p (1+x)^q \, dx$$

$$\int x(1+x^p)^q \, dx$$

What methods of integration will work in each case? Why?

A task like this could be mathematically challenging for many practising mathematics teachers. Perhaps you do not teach calculus to the learners that you work with, but whatever mathematical level you teach at is likely to contain areas that make you pause for mathematical thought. These are good areas to try to develop. It doesn't have to be something particularly hard or impressive-sounding; it could be some content from the secondary curriculum. Because of their age or the country they grew up in, some mathematics teachers may never have studied topics such as transformation geometry, circle theorems, moving averages or some statistical tests, for instance. Perhaps you have never read Euclid but always wanted to? Perhaps you have heard about *Möbius bands* but would like to learn more about topology? Popular mathematics paperbacks can be a great way to start (e.g., Gardner, 2005; Stewart, 2010).

2.1.2 A learner of mathematics pedagogy

> Those who can, do; those who can't, teach; yet even those who can, can't always teach!
> Anon.

Not everyone appreciates that there is more to mathematics teaching than knowing the mathematics. *Pedagogy*, or 'the science of teaching' (Simon, 1981), involves understanding how learners learn. Sometimes parents assume that anyone with a knowledge of mathematics will make an effective teacher, and occasionally courses designed to help teachers to teach 'hard mathematics' (e.g., post-16) focus solely on the mathematical content, leaving teachers uncertain how to help learners engage with that content for themselves. Accepting the fuzziness of the social sciences can be hard for a mathematics teacher brought up on strict deductive logic.

Task Uncle

We know that he's struggling a bit with his maths at the moment, but his uncle has a maths degree, so he's going to help him with it.

Parent

What might be the possible benefits or disadvantages of this?

It would be dangerous to over-privilege the teaching profession by implying that no one could possibly be able to help a child with their learning unless they have a teaching qualification. Clearly, many people possess the patience, interest and understanding to assist a learner's mathematical development. A learner might relate well to a trusted member of the family who has the time to spend with them that their busy teacher might not appear to have (see chapter 14). But it is by no means guaranteed that any mathematician will make a good teacher, even if they can remember their elementary mathematics from years ago. Pedagogy is vital if teachers are to be effective (Gore *et al.*, 2006).

When reflecting on their own learning of mathematics at university, mathematics teachers sometimes report being very disappointed with the quality of their lecturers, even though they may have had outstanding subject knowledge. It is clearly not enough just to be able to do the sums yourself! An effective teacher will be aware of common difficulties and will have ideas of questions and tasks that may help learners to work things out for themselves. They will also know the sorts of ideas that are likely to be taught at particular ages. A learner who was meeting trigonometry for the first time in Year 9 was puzzled when his father tried to help him but assumed that the angles would be measured in radians rather than in degrees. Family members sometimes call on methods they recall (or partially recall) being taught when they were at school, and these may not match the child's experience. This might lead to a valuable widening of experience for the learner, but on the other hand could potentially lead to clashes.

Shulman (1986) refers to *pedagogical content knowledge* as that:

> which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for teaching* ... the most useful forms of representation ... the most

> powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others.
>
> (p. 9)

Every teacher, whether they regard themselves as a 'natural born teacher' or not (Grambs, 1952; Whitbecka, 2000), can improve in the ways they interact with learners and become more effective. It is the primary aim of this book to assist with this process.

Sometimes, very knowledgeable mathematics teachers can intimidate learners (and sometimes colleagues!) with what they know, and may find it harder to be sympathetic to difficulties that they cannot remember ever having had themselves. It may be the case that some mathematics teachers have grown to love the subject because they have learned that it is an arena in which they can show off. From an early age they may have derived pleasure from the kudos of being able to do a subject that is widely perceived as difficult. Some of the stereotypical traits of a mathematician include impatience and a desire for efficiency, which can lead to insensitivity when learners are taking their time over something or 'going all around the houses' to find a cumbersome solution. This is not inevitable, but it can be helpful to acknowledge the danger.

2.1.3 A learner of the learners that you teach

> *When you observe a maths lesson, watch the* pupils *more than the teacher.*
>
> Advice from a mathematics teacher

The most significant resource that you have at your disposal for developing as a mathematics teacher is the learners that you teach. Immersing yourself in the classroom, and training yourself to be attentive to what is happening, is one of the best ways to develop your practice. In chapter 10, we will consider specific strategies for becoming a better listener, with the development of the learner in mind, but here the focus is on the immediate benefits for the teacher. Learners are human beings, and getting to know them as people can be one of the most rewarding aspects of being a teacher.

Task Zero

At the end of a lesson, a learner stayed behind to ask a question:

> This is probably a really silly question, but I've never actually been sure: is zero actually a number?

How might you respond to this question?

Every mathematics teacher will have anecdotes of incidents that have happened in the classroom and that have stuck in their mind. Sometimes such stories can take on a normative role, being repeated so often that they become canonical and come to symbolize

for that teacher some important aspects of learning mathematics. In discussion with colleagues, it becomes apparent where experiences overlap and where something seems to be unique and idiosyncratic. Both kinds of incident can be opportunities for learning if teachers take the time to reflect on them (see pages 22–23).

Task Learners

Think of a particular class that you teach.
What do you know about them as people?
What do you know about them mathematically?
What *don't* you know, and how could you find it out?
Might it help you to teach mathematics to them more effectively?

Mastering learners' names is an important part of relating to them as individuals, but it is only the start. Knowing how learners tend to think, what their interests and difficulties are and how to engage productively with them is vital. It is possible for teachers to assume that learners are just like themselves (or themselves at that age) and to suppose that what interests or appeals to the teacher will interest the learners, which may not always be the case! An opposite danger is *othering* the learners, regarding them as so different that they cannot possibly appreciate mathematics in the way that the teacher does. This can lead to a *dumbing down* of lesson content ('pearls before swine') which fails to take seriously learners' mathematical potential.

2.2 Being self-reflective

> I feel like I've got into a rut; my lessons are less interesting now than when I started teaching.
>
> Mathematics teacher

One of the dangers of experience is that we become a prisoner of our previous choices. Because we have got used to doing something in a certain way, it feels comfortable and right whenever we do it, and the more we acquiesce the more embedded that behaviour becomes, so that the idea of changing our practice is perceived as risky and threatening. Being more aware of the choices that you make *in the moment* can be a great way of recovering control and allowing your practice to develop in different directions. Schön (1983) describes the *reflective practitioner* as one who can think and act at the same time, and so deal professionally with new and difficult situations. Teachers frequently know more than they think they know; such *tacit knowledge*, acquired through experience, may be hard to put into words, but contributes to their behaviour in the classroom. The process of *reflection-in-action* seeks to make this knowledge explicit by *problematizing* things which had previously been taken for granted. One way to do this is by naming them (Pollard, 2006).

Task Critical incident

Think of a mathematical incident from a lesson that you have taught or observed.
It could be something said or done by the teacher or by a learner.
What is the most important feature of this incident for you? What do you think has made it stick in your mind?
What do you take away from this incident?

Mason's (2001) *discipline of noticing* involves developing one's sensitivity (*mindfulness*) so as to be aware of alternative possibilities when they are needed in practice. It can be very valuable to keep a *journal* (or use a voice recorder) to make what Mason calls *brief-but-vivid* descriptions of incidents or ideas that occur during mathematics lessons. It is easy to lose a thought or forget the details of a situation, and so miss an opportunity to reflect. Even a quick scribble made at the time can be sufficient to re-enter mentally, at a later moment, what it was that happened, and perhaps turn it into a piece of writing such as a blog. Mason advises distinguishing between an *account-of* an incident, which is objective and presents the facts (e.g., direct speech) in a neutral and unemotional manner, and an *account-for* the incident, which includes subjective judgements, justifications and attempts to explain what took place. By holding back initially on loaded language, alternative interpretations may be more easily considered and preconceived ideas challenged.

Task Opportunity

Have you ever realized in the classroom, just too late, that you have missed a great opportunity to do something?
What did it feel like?
Why do you think that you realized only *afterwards*?

Perhaps a learner makes an interesting comment and you say 'That's interesting' but are unsure how to develop it, and the moment slips away. Perhaps you interrupted a classroom discussion to say something and then reflected that if you had waited a little then a learner, rather than you, might have said something similar – or better. Being reflective involves developing your self-awareness to the point where it becomes *transformative*. When teachers can cultivate the ability to *think while acting*, they open up more opportunities to do something different. This involves developing an *internal supervisor* (Casement, 1990) who holds back from getting totally involved in what you are doing so as to retain a degree of objectivity. This *alter ego* can act as a *witness* to what is going on, noticing things and reporting back later, but can also intervene in the moment and change a particular course of action into a more useful one.

Task Triangle areas

A triangle has sides of length 4, 5 and *x*.
What is the maximum possible area of the triangle? Why?

Standing back and watching yourself at work can be an enlightening process. If you did this task, did you notice how you went about it? (There are some comments at the end of the chapter.) Noting explicitly what you are doing mathematically can open up different possibilities. Likewise, noticing an action in the classroom as you do it, and even before you do it, right down to the level of particular words and phrases and gestures, can enable you to take some control. The practice of talking about what is going on (*meta-commenting*), for instance in a blog or journal, focuses attention on the details of the situation.

2.3 Sharing experiences

> If you close your door, you keep *out* more than you keep *in*.
>
> Anon.

Despite working with people all day long, a teacher can sometimes find their profession a rather isolated one. Once you close your classroom door, you may feel cut off from other adults, and if things go wrong you may feel unable to talk to anyone about it – especially if they all seem incredibly busy, successful or weighed down with their own problems. However, as Pollard (2006: 21) comments, 'The value of engaging in reflective activity is almost always enhanced if it can be carried out in association with other colleagues'.

Task Contact time

What contacts do you typically have with colleagues during the day?
What opportunities are there to develop professionally from these?

All teachers need meaningful contact with colleagues during the working day, and that means more than formal meetings and the exchange of information about learners. Schools which invest in a large good-quality staffroom and entice teachers out of their departments at break time by providing decent cheap refreshments are very wise, because they encourage teachers to spend supportive time together. This can be a very pleasant way of developing your practice. There can be a great deal of common ground with colleagues who teach different subjects, and informal co-evaluation of lessons can easily take place over a cup of tea without either teacher recognizing what is happening. Of course,

you cannot talk about mathematics education all day long, and it is very important to unwind and relax in between lessons. In schools where administrative demands are deliberately kept as low as possible and teachers' non-contact time is fiercely protected, teachers are more likely to be able to find the time to 'talk shop' as well as chat about life during their free time.

Many schools have systems to encourage colleagues to share resources. Sharing your ideas will help *you* as well as your colleagues: hoarding is harmful. If you try to cling to the materials that you have created and not let others use them, you are probably less likely to think of new ideas – a river that is dammed will turn stagnant. Magicians belong to a 'magic circle' in which they are sworn to secrecy, so that the details of their best tricks are not revealed to the public. Yet some members have deliberately sought to spill the beans on the grounds that it is good for everyone in the long run, because it forces magicians to innovate rather than recycling all the old ideas *ad nauseam*. It is now easy for mathematics teachers at different schools to share ideas through professional associations such as the *Association of Teachers of Mathematics* (www.atm.org.uk), the *Mathematical Association* (www.m-a.org.uk) and the online community at the *Times Educational Supplement* (www.tes.co.uk). The associations produce journals containing thought-provoking articles, and reading what others have written, and contributing yourself, is an excellent way to develop your thinking about mathematics teaching. The associations also run annual conferences, which are the highlight of the professional year for many mathematics teachers; they offer you the chance to 'recharge your batteries' and enjoy working on mathematics yourself and with interested colleagues and discussing what goes on in the classroom.

In your non-school life, try to notice when you learn something. If I have a cookery lesson or listen to a tour guide on holiday, I find myself being aware of how they do their job and interested in the learning process *per se*. If you have the opportunity to work with young people in a voluntary capacity, such as youth worker, or with a sports team, you may gain a great deal of understanding that will help within the school environment. You might watch some teenage television programmes, listen to popular music or look at magazines and computer games that appeal to that age group, if you haven't done so since you were their age. These things will not necessarily enter directly into your mathematics lessons – although sometimes you might be able to find connections – but if you are interested in working with young people then it is natural to want to understand as much as possible about how the world looks to them. If you are a form tutor, then it will be natural to talk to your learners about their lives and sometimes to watch them play a sports match or perform on stage or in a competition.

No one can tell you how to be a mathematics teacher, and you may find yourself disagreeing sometimes with colleagues that you respect greatly, because although what they do may work very well for them, it just doesn't for you. But it is sensible to hesitate before ruling out what anyone has to say. Whatever someone's experiences may be, try to find something to learn – even if it is why you *don't* want to take on a certain approach yourself. If you can observe and talk to as wide a range of mathematics teachers as possible, you will be able to maximize the diversity of your experiences. If you hear about teaching at another school that is quite different from what you are familiar with, try to get permission to spend a day or two there 'taking the temperature' – there may be ideas that you can take away, and maybe something that you can contribute to what they do.

Task Summary task

Common sense is the collection of prejudices acquired by age eighteen.
Albert Einstein

Is there a particular area of mathematics that you would like to learn or re-learn? How could you do this?
What are the chief ways in which you intend to develop as a mathematics teacher?
What will help you to do that? What may hinder the process?
What things will help you to make *different* 'mistakes' rather than the same ones over and over again?

Comments on mathematical tasks

Task: Dots

Most people can *subitize* (see at a glance how many there are) up to a small number of dots, but not this many. There are different ways of dividing up the dots to make them quicker to count, perhaps using your knowledge of *triangle numbers*, for instance. Generalizing will lead to different but equivalent algebraic expressions for the number of dots in the *n*th drawing. One elegant approach is to see the dots as three faces of a $5 \times 5 \times 5$ array of dots, the three faces meeting at the centre of the picture (Figure 2.1). Envisioning the visible dots as those on the surface means that there will be $5^3 - 4^3 = 61$, which is the 5th *centred hexagonal number* (or *hex number*).

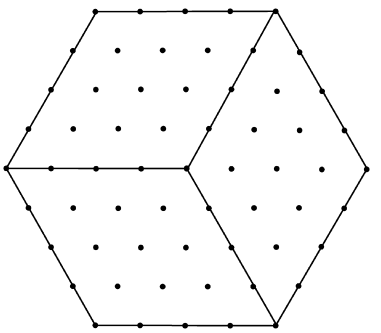


Figure 2.1 Dots on a cube

Task: Integration

One way to begin this task is with familiar 'special cases', such as $\int x(1+x^2)^q dx = \frac{(1+x^2)^{q+1}}{2(q+1)} + c$, where $q \neq -1$ (perhaps with q as a specific value). Several different methods of integration,

such as substitution and integration by parts, may be used. Did you think about p and q not necessarily being positive and not necessarily being integers?

Task: Triangle areas

Did you reach for a trusty formula such as Area $= \frac{1}{2}ab\sin C$, setting $a = 4$, $b = 5$ and $C = 90°$, so as to maximize the area? Or did you think more visually? Perhaps you envisaged the angle between the two sides changing – were you sure straightaway that a right angle led to the maximum area? Or perhaps you worried about calculating the third side – maybe the values of 4 and 5 led you to think of a 3-4-5 right-angled triangle? Perhaps you inscribed your triangle in a circle?

Further reading

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