

**SINGULAR PERTURBATIONS AND TIME SCALES
IN CONTROL THEORIES AND APPLICATIONS:
AN OVERVIEW 2002-2012**

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Abstract. This paper presents an **overview** of singular perturbations and time scales (SPaTS) in control theory and applications during the period 2002-2012. The previous overviews/surveys were provided for the period up to 1976 [241], 1976-1983 [377], and 1984-2001 [312]. Due to the limitations on the scope and space, this is in no way intended to be an exhaustive survey on the topic.

Key Words. Singular perturbation, time scale, control system, order reduction, control theories and applications.

1. Introduction

From the perspective of systems and control, Kokotovic and Sannuti [243, 384, 385] were the first to explore the application of the theory of singular perturbations to continuous-time optimal control, both open-loop formulation leading to two-point boundary value problem [243] and closed-loop formulation leading to the matrix Riccati equation [385]. The methodology of singular perturbations and time-scales (SPaTS), “gifted” with the remedial features of both dimensional reduction and stiffness relief, is considered as a “boon” to systems and control engineers. Thus the goal of SPaTS techniques is to reduce and simplify the software and hardware implementation.

The technique has now attained a high level of maturity in the theory of continuous-time and discrete-time control systems described by ordinary differential and difference equations, respectively. The growth of research activity in the field of SPaTS resulted in the publication of excellent survey papers [438, 338, 55, 254, 156, 241, 439, 236, 377, 237, 316, 313, 209, 251, 310, 60, 440, 314, 311, 315, 312, 204, 206], reports and proceedings of special conferences [242, 122, 13]. Also, see research monographs and books (including the general area of singular perturbation theory) [126, 119, 463, 217, 95, 120, 320, 441, 339, 121, 288, 292, 433, 226, 321, 94, 191, 73, 317, 432, 322, 411, 239, 240, 169, 238, 32, 309, 131, 146, 253, 180, 306, 340, 54, 447, 147, 2, 442, 227, 230, 31, 387, 7, 144, 410, 481, 182, 402, 491, 408], encyclopedia [409] and control handbook [229].

This paper presents an overview of singular perturbations and time scales (SPaTS) in control theory and applications during the period 2002-2012. The previous

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overviews and/or surveys were provided for the periods up to 1976 [241], 1976-1983 [377], and 1984-2001 [312]. Due to the limitations on the scope and space, this is in no way intended to be an exhaustive survey on the topic.

Those readers who do not have some basic background in SPaTS, need to refer to any of the above references, in particular, [240, 309, 147, 312].

2. Modeling

About the basic singular perturbation modeling background, please refer to the reference [312]. Besides, a new unified modeling method is developed using δ -operators in [258] and a bond graph model is presented in an integral causality assignment in [163]. Liyu Cao introduced a new reduced-order model which is based on the actual value of ϵ [62].

In SPSs, fast subsystems produce a limiting system for the slow subsystem. It is shown in [168] that if the flows produced by the unperturbed fast subsystems are “chain transitive”, the limiting system can be approximated. And “the reachable sets are contained in the reachable sets of the slightly inflated singularly perturbed system”.

2.1. Linear Time Varying (LTV) Systems. In [484] a larger bound on ϵ is provided which is a gauge on the validity of the Chang transformation. Gauss-Seidel iteration method is used to investigate the exponential stability of singularly perturbed LTV systems in [75], and a method is proposed to compute the upper bound of ϵ under which the system is exponential stable.

2.2. Nonlinear Control Systems. A bond graph model for a nonlinear singularly perturbed system is presented [22]. A new discretization scheme for two-time-scale nonlinear continuous-time systems is proposed based on Euler’s methodology in [35].

For singularly perturbed Hodgkin-Huxley system, Neumann boundary conditions are given from [40]. In [92] gain scheduling control is designed for a nonlinear singularly perturbed time-varying system. In [271], a class of nonlinear memoryless controllers is synthesized for a class of imperfectly known nonlinear SPSs with discrete and distributed delays.

In [319] a holographic explanation is given to show how the renormalization group approach to singular perturbations in non-linear differential equations proposed by Chen, Gold-enfeld and Oono is indeed equivalent to a renormalization group method in quantum field theories proposed by Gell-Mann and Low via AdS/CFT correspondence.

2.3. Hybrid Control Systems. If a singularly perturbed system includes both the continuous and discrete states, or both the continuous-time and discrete-time, or both the time driven and event driven properties, then it is called a singularly perturbed hybrid systems.

The stability of singularly perturbed hybrid systems is analyzed in [383, 76, 457]. The oscillation conditions of a second-order singularly perturbed hybrid linear delay dynamic equation are discussed on different time scales in [125].

In [456, 457] the solutions of a class of singularly perturbed hybrid linear delay dynamic equations are discussed.

In [171] singular perturbation theory is used to decompose a hybrid system and the global bifurcations of the forced van der Pol equation are studied based on the reduced systems. Similarly, in [390, 490, 97] singular perturbation is used to deal with hybrid systems.

Also, see [489, 259] for further results on this topic.

3. Boundary Value Problem (BVP)

Boundary value problem of SPSs has attained much attention because one of the boundary conditions has to be sacrificed in the process of degeneration.

A survey [247] presents computational techniques for solving singularly perturbed BVP developed by numerous researchers between 2000 and 2005. Also see [437, 208, 362, 177] for some more results regarding numerical algorithms of solving BVPs.

New approaches are developed to find solutions of singularly perturbed BVP [50, 296, 295, 371, 363, 11, 197]. A Dirichlet problem is studied for a singularly perturbed parabolic reaction-diffusion equation in [403, 404].

In [203, 205, 200] difference schemes for singularly perturbed two point BVP are derived using spline in compression on non-uniform mesh.

Methods for solving reaction-diffusion singularly perturbed BVPs are studied in [250, 449]. In [49] for a singularly perturbed reaction-diffusion problem, the well known a priori Bakhvalov and Shishkin meshes are compared with the adaptive mesh based on the a posteriori dual error estimators.

The proof of the existence of a positive solution of singular discrete third-order BVP with mixed boundary conditions is given in [108]. For some singular perturbation problems which consist of the fourth order differential equations in [386], sufficient conditions are obtained such that the boundary layer can be ignored.

Also see [207, 24, 293, 360, 508] for more works on linear singularly perturbed BVP.

In [51, 53, 59, 246, 106, 66], methods to solve non-linear BVPs are studied and the solutions are analyzed.

A rationalization of the singularly perturbed continuous system of single and multiple time delays is presented by setting a delay coefficient in [397].

The author of [352] obtained an upper bound, in the spirit of Γ lim sup, by multidimensional profiles, for some classes of singular perturbation problems.

4. Time Scale Analysis

In [113], necessary and sufficient conditions for the existence of uniform exponential stability are derived of LTI systems on arbitrary time scales. In [382] a novel time-frequency method to analyze the phase-locked loops is presented. Singular perturbation method is used for diagnosability of linear two-time scale systems in [162] and reduction of the order of unstable linear time invariant systems is done in [467].

In [188], considering a class of second order nonlinear dynamic equations on time scales, a condition is developed that ensures the existence and uniqueness of solutions.

Also see [353, 337, 346] for works on two-time-scale discrete-time systems.

Further, for multiparameter (multi-time-scale) deterministic and stochastic systems, we decompose a full-order system with several small parameters into one low-order slow subsystem and several low-order fast subsystems. See [33, 290, 354, 44] for recent results on multi-time-scale method.

Multi-time-scale method is applied to the network of livestock movements and the dynamics of diseases [214], fractal dynamics in physiology [161], a small set of plant, animal, and abiotic processes structure ecosystems [181].

5. Stability Analysis

5.1. Stability Analysis of Linear Singularly Perturbed Systems. The stability and stabilization problems of continuous-time linear SPSs are studied in [133, 47, 172, 278]. For singularly perturbed Stokes problem, a new stabilized finite element method is developed in [81].

In [116] considering two-time scale systems, an explicit state-space solution of the robust stabilization problem is presented based on Riccati equations. Also the robust stability problem of uncertain two-time-scale systems is studied using a state transformation and Lyapunov theory in [393, 394].

And the authors of [510] studied global exponential stability of singularly perturbed descriptor systems with nonlinear perturbation using fixed-point principle and LMI. In [213] the exponential stability of SPS with both time delay and uncertainties is investigated. In [117] the asymptotic stability of the stability radius is described as ϵ tends to zero. Considering impulsive SPSs under nonlinear perturbation, a sufficient condition that ensures robust exponential stability for sufficiently small ϵ is derived in [77].

Stability bound problems are studied for multi-parameter singularly perturbed time-delay systems [91], linear SPSs [63], discrete multiple time-delay SPSs [89], SPSs with nonlinear uncertainties [511], and nonstandard SPS with time-varying delay [425].

D-stability problem is studied for discrete-time singularly perturbed systems (DSPS) in [186] where a system is called D-stability if the poles of the system are within the specific disk $D(a,r)$ centered at $(a,0)$ with radius r , in which $|a| + r < 1$.

[279] and [173] are both about switched singularly perturbed systems. In [279] the relationship between the stability of subsystems and that of original system is studied for switched linear SPS. In addition, a state-feedback controller is designed. In [173], stability of the planar linear switched SPS is analyzed. Further, stability of the switched linear DSPS is studied by Ivan Mallocci in 2010 using LMIs and switched quadratic Lyapunov functions[280].

In [430], singularly perturbation theory is used to study the input-to-state stability (ISS) of general systems. Assuming the system can be separated to slow and fast subsystems, “the main results establish that if the boundary layer and averaged systems are ISS then the ISS bounds also hold for the actual system with an offset that converges to zero with the parameter that characterizes the separation of time-scales”.

Singular perturbation approach is applied for time-domain assessment of the Phase Margin (PM) of an SISO LTI system, whose fast loop system is considered as a singular perturbation with a singular perturbation (time-scale separation) parameter ϵ in [513]. A bijective relationship between the Singular Perturbation Margin (SPM) ϵ_{max} and the PM of the nominal (slow) system is revealed as well as the phase of the fast system.

5.2. Stability Analysis of Nonlinear Singularly Perturbed Systems. Here, we review the SPaTS methodology as applicable to nonlinear systems and the related stability problems.

Robust regulation of a class of nonlinear SPS, is considered via nonlinear H_∞ approach in [10]. See [9, 347] for more results regarding robust stability of nonlinear SPSs.

Exponential stability condition of nonlinear SPS with uncertainties which has upper norm bounds for enough small ϵ is developed and a stabilizing controller is

proposed in [414]. Also exponential stability of non-standard nonlinear SPSs is studied in [93, 345].

Using singular perturbation theory, sufficient conditions of the global asymptotic stability of a class of scalar nonlinear difference equations is given [192]. The absolute stability problem for Lur'e SPS with multiple nonlinearities is studied in [478, 479].

A new concept of mesh stability is proposed for a class of interconnected nonlinear systems in [344] and a set of sufficient conditions of mesh stability are derived. Two different definitions of semiglobal practical external stability are discussed in [12].

Also see [389, 427, 375, 71] for more related works on stability of nonlinear SPSs.

Besides stability problem, more properties of nonlinear SPSs are analyzed during 2002-2012. In [42] an averaged system is constructed to approximate the slow dynamics of a two time scale nonlinear stochastic control system and the approximation is shown valid. Similarly, in [143] an averaging technique is developed.

The limit occupational measures set is presented in terms of the vector function defining the system's dynamics in [142]. In [202] an exponentially fitted difference scheme using cubic splines for a singularly perturbed ordinary differential equation is derived.

The proof of the existence of critical points with semi-stiff boundary conditions for singular perturbation problems in simply connected planar domains is given in [256]. 'Two-stages' strategy used with singular perturbations is extended to compute a balancing form of nonlinear singularly perturbed system in [110, 112].

In [15, 159, 166, 157], order reduction approaches to singularly perturbed nonlinear systems are presented.

And about the solutions of the nonlinear SPS, the conditions of existence are presented in [464, 52, 218], estimate solutions are developed in [448].

Singular Perturbation Margin (SPM) is proposed in term of ϵ in [485]. The SPM relationship between LTI and NLTI systems at the equilibrium is presented. In [286] a new concept of the point-wise eigenvalues and eigenvectors are defined and exploited "as an indicator of the local rate of change of the state of a nonlinear system".

The problem of passivity and passification for a class of nonlinear SPSs is studied via neural network in [465]. The upper bound of perturbation parameter can be obtained by solving algebra inequalities, and the proposed controller can make the singularly perturbed nonlinear system passive. Compared with [465], time-varying delays and polytopic uncertainties are added to the singularly perturbed nonlinear systems in [466]. And the results are generalized.

In [303], singular perturbation technique is used to deal with nonminimum-phase multiple-input-multiple-output (MIMO) nonlinear systems. In order to design oscillation controller in non-linear systems, singular perturbation method is used to analyze the stability of high order systems [61].

6. Observers

In [160] observability of singularly perturbed linear time-dependent differential systems with distributed time-delays in state variables is studied. A new recursive algorithm for solving the multiparameter algebraic Riccati equations to obtain the optimal Kalman filter for multiparameter SPS is developed in [300].

Singular perturbation theory is used to study the well posedness of observer-based fault detection filters in [334], observer design for second order mechanical systems [109], and sliding mode observer design [291].

See [470, 165] for more works related to observers of SPSs.

7. SPaTS in Optimal Control

7.1. Open-Loop Optimal Control. Hiroaki Mukaidani revised Kleinman algorithm based on Newton-Kantorovich theorem to solve algebraic Riccati equation of SPSs, and its quadratic convergence property is proved [257]. In [486], singular perturbation method is used to get a reduced system based on which optimal poles are found.

Optimal control problems are studied for discrete-time and continuous-time SPSs in [123, 58, 376, 101]. [145] is a view on optimal control of linear SPSs and applications.

7.2. Closed-Loop Optimal Control. The closed-loop optimal control has some very elegant results for singularly perturbed systems.

See [153] for study on LQ (linear quadratic) decentralized pole location for SPS, and composite LQ control for SPSs [263].

In SPS, “the variational limit, as the ratio of time scales grows, is best depicted as a trajectory in a probability measures space”. In [16], the variational limit in the form of the Pontryag in Maximum Principle is presented. Also its relationship with the Maximum Principle of the system is discussed.

See [400, 14, 304, 219, 379, 487, 355, 327] for more works on closed-loop optimal control problems of different kinds of linear continuous-time SPSs.

A generalized approach from continuous time system to discrete systems of designing a state feedback controller to get a specified insensitivity of the closed-loop trajectory by the singularly perturbed unified system is developed in [401] and necessary conditions for optimality are derived.

Also, in [36, 231, 25] optimal control for Discrete Singularly Perturbed Systems (DSPSs) is studied.

In [299, 301], “the linear quadratic Nash games for infinite horizon multiparameter SPS with uncertain singular perturbation parameters are discussed”. Based on successive approximation, a construction of high-order approximations to a strategy that guarantees a desired performance level is presented, which improves the cost performance.

8. Other Control Problems

8.1. Robust Control problems. The robust control is studied for SPSs with nonlinear uncertainties in [493], with delay in [469]. The time-delay effect on the robust stabilization of an uncertain SPS via a networked feedback is presented when the time delay is smaller than the sampling time in [459].

Grammel considered the nonlinear SPS with small time delays in the slow variables in [167], and sufficient conditions of exponential stability of the slow subsystem being robust is established using averages of the fast variables.

The author of [70] presented a three-time-scale redesign which stabilizes nonlinear systems with input uncertainties and recovers the nominal closed loop trajectories. This is achieved by designing a high filter which estimates the uncertainty over a fast time scale, and then forcing them to converge to the nominal input manifold by another set of fast filters.

8.2. Fuzzy Control Problems. In [499], delta operator is used to construct a fuzzy singular perturbed unified model which applies to both continuous-time domain and discrete-time domain and the robust control is presented for the proposed model framework. Singular perturbation technique is used for order reduction of linear complex systems described by TSK fuzzy models in [46].

In [185], a robust fuzzy controller is designed for nonlinear multiple time-delay SPSs and in [18] an H_∞ fuzzy controller is designed for a class of nonlinear SPSs, which can be used for both standard and unstandard nonlinear SPSs.

Similar results exist for nonlinear singularly perturbed Takagi-Sugeno (TS) fuzzy models, designing fuzzy H_∞ filter, H_∞ output feedback controller and composite fuzzy controllers in [17, 21, 20, 19, 261, 323].

Also see [124] for a fuzzy logic algorithm to optimize sliding surface parameter. In [273] based on the stability analysis of both continuous-time and discrete-time fuzzy SPS, stabilizing feedback controllers are designed separately.

A novel high gain observer-based decentralised indirect adaptive fuzzy controller is developed for a class of uncertain affine large-scale nonlinear systems in [187]. Multi-objective control which consists of H_∞ control, pole placement and singular perturbation bound design for T-S fuzzy SPS is presented in [480]. Also in [498] the decentralized multi-objective robust control problem is discussed for interconnected fuzzy singular perturbed model (FSPM) with multiple perturbation parameters.

8.3. Network Control Problems.

For some results on the topic of network control problems, see [27, 305, 372, 276] for singular perturbation theory applications in network systems.

A two-time-scale plant is analyzed in [494] whose sensor is connected to a linear controller/actuator via a network.

In addition, the model-based networked control is studied in [496, 458] for SPS and SPS with uncertainties. The neural network-based control and observer design for a class of nonlinear SPSs are studied with guaranteed H_∞ control performance in [269].

8.4. H_∞/H_2 control problems. New methods are presented to solve the H_2/H_∞ control problems for continuous-time SPS in [302, 128].

Also, H_∞ controllers are designed using different approaches for multiparameter SPSs, SPS with norm-bounded uncertainties, nonlinear SPS of TS fuzzy model, and uncertain SPS in [477, 431, 134, 135, 482, 270].

Robust multi-objective H_∞ control is studied for linear two-time scale systems in which fast dynamics are assumed of norm-boundedness in [220]. H_∞ control is studied using singular perturbation theory for inclusion nonlinear systems in [294].

For standard DSPS with polytopic uncertainties, state feedback H_∞ controller is designed by using LMI in two ways in [114]. In [105] a mixed H_2/H_∞ linear state variable feedback suboptimal controller is designed for a DSPS based on reduced order slow and fast subsystems. See [115, 272] for more results regarding to H_∞ control problem of DSPS.

See [170, 483] for more works on H_∞/H_2 control and SPaTS.

8.5. Other Control Problems. For two-time-scale systems, an approximate controller is designed in [398], ergodic control is considered in [43].

Considering linear SPS with time-delay in [88], reduced subsystems are obtained via singular perturbation techniques and the relationship of controller and observer design between the original and the subsystems obtained is presented.

In [154] composite and reduced input bounded controller are designed for linear SPS and constructive geometric conditions are proposed to drive a stabilizing controller in both composite and reduced contexts. For a class of interconnected SPSs in [335], a controller is designed that reduces the trajectory sensitivity to small feedback delays using the singular perturbation and sensitivity theories.

Also, sliding-mode control is studied of SPSs in [152, 139, 3]. Observer-based feedback controller is designed for continuous-time SPS in [267].

Besides, guaranteed cost control, composite control, passivity-based control, adaptive control, switched output feedback control, admissible control, and variable structure control are studied for different kinds of SPSs in [264, 504, 90, 151, 497, 268, 5, 495, 41, 329].

Singular perturbation theory is applied to other control problems. See [211, 381, 190, 399, 415, 333, 328, 136, 176] for applications of singular perturbation approach to study dynamical feedback control, observer-based control, high-gain feedback control, variable structure control, indirect adaptive control, sliding mode control, cost control, and tracking control problems for various systems.

Singular perturbation theory is used to derive the structure of controllability and observability energy functions of bilinear SPS in [111].

About the control problem of nonlinear SPSs, fault tolerant control, optimal control, closed-loop composite control, PI control, feedback control, PID control, adaptive control, and tracking control are studied in [512, 232, 233, 452, 500, 501, 502, 406, 407] respectively. Model predictive control problem is studied in [79, 78, 468, 331].

9. Numerical Algorithm for SPSs

Because of the stiffness and high order property of the SPSs, it is difficult to get the analytical solutions for singularly perturbed equations, therefore numerical algorithms are developed and applied to SPSs.

Computational algorithms are developed to solve self-adjoint singularly perturbed BVPs in [29, 221]. Please refer to [436, 82, 392, 4, 34, 342, 503] for more results about numerical algorithms to solve singularly perturbed BVPs.

The author of [378] described numerical methods for solving stiff initial-value problems (IVP) using one-step schemes of exponential type.

A computational algorithm is developed to get the solution of a singularly perturbed differential-difference equation with turning points which can result in boundary or interior layer in [359]. In [104, 102, 103], an efficient numerical scheme is proposed to solve SPSs of Robin type reaction-diffusion problems.

More algebraic and numerical approaches are presented to get the solutions of singular perturbation problems in [201, 429, 175, 199, 210, 298, 507].

Nonstandard finite difference method is introduced to solve the singularly perturbed differential equations by [349]. Works [248, 249] present a survey of the most effective computational techniques for solving singularly perturbed partial differential equations. A computational method is presented for solving singularly perturbed delay differential equations with negative shift whose solution has boundary layer [129].

The Benoit's theorem is extended in [158] for the generic existence of solutions of SPSs of dimension three with one fast variable to those of dimension four. For a singularly perturbed nonlinear Robin problem in a periodically perforated domain,

[107] proved the existence of a family of solutions for ϵ sufficiently small. Considering the Helmholtz equation, $\epsilon^2 u_{xx} - u = f(x)$ where ϵ is small, two methods are developed to get the solution in [48].

9.1. Asymptotic Approximation for Solutions of SPSs. Via numerical methods, we obtain the asymptotic approximations to give the qualitative behavior of the solutions. The details of obtaining the approximated solutions are given in [463, 441, 309].

The asymptotic expression of a kind of vector singularly perturbed delay-differential equations in [451] is constructed and the uniform validity of asymptotic solution is also proved. Asymptotic approximations of the solutions for different classes of SPSs are also obtained in [471, 287, 184].

In [252], asymptotic expansion of solutions of optimal control problems for singularly perturbed systems (SPS) are constructed. For Markov random process, asymptotic expansions and probability distributions are studied using SPaTS theory in [6, 326, 491].

Also explicit bounds on the convergence rate of the trajectories to the slow manifold and on the asymptotic error between the trajectories of the SPS and those of the reduced system are obtained in [444, 445].

10. Applications

10.1. Aerospace. The theory of SPaTS has its roots in fluid dynamics and naturally found its wide applications in the area of aerospace systems. See a survey [361] on applications of SPaTS on aerospace.

See [422, 357, 285] for applications of singular perturbation to digital flight control systems.

For two-time-scale aircraft dynamics systems, see interesting applications of SPaTS in [164, 398]. Singular perturbation approach is utilized to aircraft systems in [473, 127] for multi-time-scale decomposition.

In [45], considering a class of nonlinear systems actuated by actuators whose actuator dynamics are assumed fast, baseline controller is designed. Singular perturbation is applied to show that the closed loop system achieves the control objective.

A postbuckling analysis is presented for nano-composite cylindrical shells reinforced by single-walled carbon nano-tubes subjected to combined axial and radial mechanical loads in thermal environment in [396], and a boundary layer theory and associated singular perturbation technique are employed to determine the buckling loads and postbuckling equilibrium paths.

Also, see [183, 118, 358] for applications of singular perturbation and time-scale methodologies to missile systems.

10.2. Mechanical Systems.

Another interesting area of the application of SPaTS is mechanical dynamics and control

For a multi-link flexible robot with uncertainties, an improved composite controller is designed based on singular perturbation theory in [86]. In [380] a new control strategy for flexible-joint manipulators with joint friction is proposed. The proposed controller includes two main components: a friction compensating torque, and a composite controller torque which is designed using singular perturbation theory.

Also see [84, 99, 341, 266, 348, 100, 274, 428, 424, 64, 423, 265, 85, 405, 395, 435] for more results about applications of SPaTS to flexible robots.

See [69, 283, 453, 150] for more results of SPaTS on other kinds of robots.

In [98, 462, 461], closed kinematic chains (CKC) are modeled as singular perturbation systems and the properties of validity domain, error characterization and stability are analyzed.

In [130] results on partial stability of the differential form of speed-gradient control for SPS are generalized to the case of speed-gradient control in finite form.

Based on singular perturbation techniques, direct torque control (DTC) is derived in [417] and a link between DTC and feedback linearization is presented. “An explicit relationship between DTC performance and machine characteristics has been revealed”, which can be used to improve DTC performance by designing an induction motor.

In [460] the singular perturbation formulation is compared to control based on input-output linearization (IOL) and advantages and disadvantages of each method are described.

Traveling wave solutions of viscous conservation laws are studied in [1]. The eigenvalue problem corresponding to the linearization around a viscous shock wave is viewed as a singularly perturbed problem, and geometric singular perturbation theory is used for the analysis of the Evans function. And the Gardner and Zumbrun results about the first derivative of the Evans function are proved at the origin.

A new coning correction algorithm, based on the singular perturbation technique, is proposed for the attitude update computation with non-ideal angular rate information [141]. Singular perturbation theory is applied to show that the Boltzmann-C-Enskog equation results in the Navier-Stokes equation for incompressible fluids together with two different Boussinesq relations and temperature fluctuation equations in [193] and the proof of a rigorous result is given in [194].

In addition to the above applications, singular perturbation theory is also applied to air-conditioning systems [364], an axially moving cable with large sag [368], a four-wheeled steering and four-wheeled drive vehicle [351], early detection systems with multiple-bottleneck links [462], harmonic drive systems [149], pneumatic vibration isolators [174], the voice coil motor [343], hypersonic vehicles [307], hydraulic systems [Longke-Wang-et-al2012], a flexible beam used in underwater exploration [308], infinite-dimensional mechanics of fluids and plasmas [492], dual-loop exhaust gas recirculation air-path systems [476], underactuated biped robots [87], hydrostatic drive or cylinder [281], single-axis rate gyro [74], bimolecular association mechanism [132], 2D thermal convection loop [443], and so on.

10.3. Electrical and Electronic Circuits and Systems. It is common to neglect dynamic saliency in synchronous machines of power systems [275]. This paper [80] is a summary which tells how to model power systems using singular perturbation approach, and neglect the fast dynamics to get a simplified power system model. The error associated with neglecting dynamic saliency is eliminated by inserting a singular perturbation into the machine model [350].

In [434, 420, 418, 419] singularly perturbation method is used for synchronous generator systems to sliding mode control. In [262], a doubly-fed induction generator is considered to design a controller based on multi-time scale theory.

For singularly perturbed relay systems [138, 137], a theorem about existence and stability of the periodic solutions is proved and an algorithm of asymptotic representation for the periodic solutions is presented using boundary layer method.

Forced singular perturbations are proposed to reduce computations of multirate strapdown terrestrial navigation algorithm in [450]. The transmission problem is

studied for the system of piezoelectricity having piecewise constant coefficients in [216].

For a class of DC-DC power converters, current-mode control problem is studied in [8]. Singular perturbation is used to separate the fast and slow states of dc-dc converters systems in [234], and a relationship between inductance, capacitance, load resistance, and loss resistance is obtained from an analysis of an approximate model. Compared with [234], discrete-time analysis is added in [235].

The papers [324, 325] studied the singular perturbation analysis and synthesis of wind energy conversion systems. SPaTS theory is used to analyze wind energy conversion systems in [195] and time-scale method and MPC are combined to control wind energy conversion systems in [509].

Also, see [374, 366, 140] for more applications of SPaTS to wind power systems.

For servomotor systems [506] a general PIV controller is taken into the original system, and the closed loop is decomposed to two subsystems using singular perturbation theory which stand for the position control loop and the high frequency RV dynamics, respectively.

Singular perturbation theory is used to design an observer of sliding mode type for the flux estimation of an induction machine in [332]. In order to provide insight into the connections between the different nodes of a power network, a method based on differential geometric control theory is obtained in [28]. In [179] circuit-averaging techniques are applied to simplify a lumped-parameter model of the cardiovascular system.

Considering the noncoherent digital delay lock loops on chip timing synchronisation in [178], the mean time to lose lock is calculated using diffusion approximation and the singular perturbation method. Loop bandwidth is optimized for the first order loop.

The stability of a large-scale power system is analyzed by Jacobian analysis based on singular perturbation in [472]. Considering the converter-interfaced wind turbines in [367], singular perturbation theory is used to decompose the system dynamics based on which a controller is designed to isolate wind-power fluctuations from the power grid.

In [454] a singularly perturbed model is developed for AIMD/RED systems with multiple bottlenecks and feedback delays, of which stability is analyzed. The delay-dependent LMI conditions for the stability are established and it is proved that sufficiently small parameters exist to guarantee the asymptotic stability of the system considered above.

In [282] singular perturbation technique is applied to the permanent magnet synchronous machine (PMSM) system. Based on the decoupled subsystems, “the control speed and the I_d current are carried out by neuro-fuzzy regulators (AN-FIS)”.

Based on the combination of PMSG and super sparse matrix converter, a novel Variable-speed wind energy generation scheme is developed for the wind energy conversion system in [488].

Emitter-coupled multivibrators are modeled and analyzed using the singular perturbation theory in [356]. A sampled-data strategy for a boundary control problem of a heat conduction system modeled as PDE is developed in [83]. Using singular perturbation theory, the reduced subsystems are presented.

Considering the power system model in [260], singular perturbation method is used to decompose the system into slow and fast subsystems and the relationship between the stability of reduced-order system and original system is analyzed.

In [474] a class of power systems with detailed excitation and power system stabiliser (PSS) controller is modeled as singular perturbation system. An LMI-based approach is developed to estimate the stability region. Considering multi-machine power systems with matched additive uncertainty and input multiplicative uncertainty in [72], several time-scale separation designs are used for robust stabilization and performance recovery.

A complementary controller is designed to improve power systems stability in [426] based on singular perturbation theory. Considering an oscillator model which is composed of a fast membrane potential dynamics and a slow recovery dynamics in [318], phase response curves (PRCs) for both the dynamics and plausibility of feedback inputs to the slow dynamics rather than the fast dynamics are shown using singular perturbation theory.

10.4. Chemical Reactions and Reactors. Two different reducing order methods are compared to singular perturbation method for chemical kinetics equations in [215]. And singular perturbation method is applied for model reduction of stiff chemical Langevin equations and chemical kinetics problems in [96] and [255]. Global Quasi-Linearization (GQL) method which is based on two-time-scale theory is presented for an automatic reduction of chemical kinetics models in [56].

Equations for the description of chemical reactions of dissociation and recombination are transformed into singularly perturbed equations in [148]. A new concept of critical simplification for chemical kinetics is proposed, which is valid in the presence of a dominant competitive reaction and critical phenomena in [475].

Singular perturbation method is used to analyze and synthesize the model of thermal explosion in a gas-droplets mixture [57], a chemical reactor and a feed effluent heat exchanger in [198].

Viewing the prompt jump approximation (PJA) of nuclear reactor dynamics as the zeroth-order approximation of an asymptotic expansion to SPS of ordinary differential equations, the equations describing its first-order approximation is derived [30].

Two-point linear controllers for binary distillation columns are designed based on singular perturbation theory in [65]. Considering a singularly perturbed convection-diffusion equation with constant coefficients in a half plane, with Dirichlet boundary conditions [225], precise pointwise bounds for the derivatives of the solution are obtained.

10.5. Biology. Models describing the biotechnical process behaviour are usually high order and nonlinear with time-varying parameters. In [223], decomposition techniques based on singular perturbation analysis or batch phase analysis are used to simplify the model. Also, it is shown that the essential initial values can be effectively obtained from the data.

Considering a host-vector model for a disease without immunity in [373], the stability of the steady states using the contracting-convex-sets technique is studied and using the geometric singular perturbation method, existence of travelling wave solutions is established.

The work of [505] shows that travelling wave solutions exist for a modified vector-disease model using the geometric singular perturbation theory. Models that incorporate local and individual interactions are introduced in [416]. In addition, epidemiological time scales are used to reduce the dimension of the model and singular perturbation methods are used to corroborate the results of time-scale approximations.

In [365] a mathematical model is proposed for the differentiation of osteoblastic and osteoclastic populations in bone, based on the differential effects of PTH, and singular perturbation theory is used to analyze the highly diversified dynamics.

Considering the model describing the growth of microalgae, the authors of [67, 68] maximized the specific growth rate of microalgae by manipulating the irradiance using singular perturbation theory.

The topic of [391] focuses on the analysis of a nonlinear dynamical model of a class of bioprocesses, and in order to obtain reduced order models, the singular perturbations method and quasi-steady state assumption are used.

A novel ion channel biosensor is modeled in [297], and singular perturbation theory is used to design an optimal input voltage to the biosensor to minimize the covariance of the estimation error.

A mechanism is proposed in bio-molecular systems to attenuate retroactivity in [196]. Using coordinating transforms and singular perturbation theory retroactivity can be arbitrarily attenuated by internal system gains. Singular perturbation theory is applied to analyze the bio-molecular feedback systems in [446].

In [244], the model is analyzed to predict the performance of the biosensor in transient and steady-state regimes. Singular perturbation is used to determine the conditions for globally uniform stability of a class of biological networks with different time-scales under parameter perturbations in [289].

Based on singular perturbation theory, dynamical system theory, and differential-algebraic equations, a mathematical framework is developed to analyze and design on-line schemes for fixed point recurrent neural learning in [369].

10.6. Other Areas.

In [413] the moisture-induced deformation in an elastic panel is modeled as singularly perturbed system, which is approximated by singular perturbation method. Compared with experimental results, the solution is analyzed based on the material of the elastic panel. Optimal climate control problem is studied for a potato storage facility to exploit the favorable weather conditions in [224]. Using singular perturbations and based on Fenichel's theorems, extensions of Hirsch's generic convergence theorem for monotone systems are studied in [455].

Considering a large-scale nonlinear network system, singular perturbation is used to decompose the states into fast and slow subsystems in [37, 38], and the validity of the reduced-model approximation is proved on the infinite time interval.

The notion of two-time-scale (TTS) distributions is introduced [336] and TTS distribution is analyzed in two different time scales. Singular perturbation is applied to choose the Page Rank factor in a bow-tie web graph in [23].

Boundary value theory is applied to analyze the gravitational-tidal evolution of planetary subsystems of the Sun in [39]. The topic of [26] is about the spectral properties of the Neumann-Laplacian on the singularly perturbed periodic quasi-cylinder. Singular perturbation theory is used to analyze the global asymptotic stability of positive equilibria of ratio-dependent, predator-prey models with stage structure for the prey in [330].

In [155], singular perturbation theory is applied to analyze the quadratic family with multiple poles. The author of [370] analyzed the Stokes flow in a singularly perturbed exterior domain. Considering static and dynamic behaviour of two-dimensional droplets in [388], an evolution equation for the droplet thickness is obtained using singular perturbation theory.

In [421], "vorticity distributions over the diffracted shock both from Lighthill's theory applicable for small bends and Sakurai and Takayama's theory applicable

for larger bends have been investigated for Mach numbers 1.80 and 1.95” using singular perturbation theory.

11. Conclusions and Future Directions

11.1. Conclusions. This paper presents an overview of singular perturbations and time scales (SPaTS) in control theory and applications during the period 2002-2012. The works on optimal control, robust control, fuzzy control, network control, H_2/H_∞ control, stability analysis, numerical algorithms and other control problems of SPSs during 2002-2012 are reviewed. In the end, applications of SPaTS theory to aerospace, mechanical, electrical and electronic systems, chemical reactions, biology and other areas are presented.

11.2. Future Directions. Some of the areas for future investigations are suggested below.

- (1) One area is to investigate singular perturbation techniques for *nonlinear* systems in general and explore the possible applications to a variety of systems such as wind energy systems and life science problems. In particular, this may lead to exploring the singularly perturbed State-Dependent Riccati Equation (SDRE) - both differential and algebraic for continuous-time and discrete-time systems with time-scale character.
- (2) Another interesting and recent area is *Model Predictive Control (MPC)* of linear and nonlinear and deterministic and stochastic Time Scale Systems [78, 79].
- (3) Also, another area worth looking is the absolute stability for uncertain Lur’e SPS based on the Lyapunov theory, and Linear Matrix Inequalities (LMIs).
- (4) Control for singularly perturbed *descriptor system* with nonlinear perturbation [510] seems an interesting area.
- (5) Control for singularly perturbed *switched linear control systems* [173].
- (6) Since time-scale character occurs so naturally in many science and engineering systems, it is worth exploring the application spectrum to areas in science (biology, economics, management, etc.) and engineering (Aerospace, Biomedical, Chemical, Electrical, Mechanical (including robotics), Nuclear Engineering, etc.).

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JULY 1996. Renormalization group and singular perturbations: Multiple scales, boundary layers, and reductive perturbation theory. With numerous examples, we illustrate its application to ordinary differential equation problems involving multiple scales, boundary layers with technically difficult asymptotic matching, and WKB analysis. renormalized perturbation theory, and the RG equation describes the long-time scale motion of the amplitude and the phase. III. The basic observation on which our approach relies is a correspondence between time in the initial-value problem and length scale (or rather the logarithm of length scale) in old theory. Thus the Cauchy data can be regarded as being analogous to the bare parameters of the old theory. Singular perturbations and time scales in control theories and applications: an overview 2002-2012. Yan zhang, D. subbaram naidu, chenxiao cai, and yun zou. Abstract. This paper presents an overview of singular perturbations and time scales (SPaTS) in control theory and applications during the period 2002-2012. The previous overviews/surveys were provided for the period up to 1976 [241], 1976-1983 [377], and 1984-2001 [312]. Due to the limitations on the scope and space, this is in no way intended to be an exhaustive survey on the topic. Key Words. Singular perturbation, time scale, control sy A singularly perturbed linear time-dependent controlled system with multiple point-wise delays and distributed delays in the state and control variables is considered. The delays are small, of order of a small positive multiplier for a part of the derivatives in the system. This multiplier is a parameter of the singular perturbation. Many well established singular perturbation theories for singularly perturbed systems require the full knowledge of system model parameters. In this paper, a new adaptive identification method for singularly perturbed nonlinear system using multi-time-scale recurrent high-order neural networks is proposed to obtain an accurate and faithful model.