# Book reviews

## Computational Partial Differential Equations: Numerical Methods and Diffpack Programming

H.P. Langtangen
Texts in Comp. Sci. and Eng., Vol 1, 2nd ed.
Springer-Verlag Berlin Heidelberg 2003
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The aim of this book, as stated in the preface is "To Teach Numerics along with Diffpack". The "target audience is students and researchers in computational sciences who need to develop computer codes for solving differential equations". I feel that the author has been successful with the stated aim, and the content is well directed to the target audience, or at least those who intend to use DIFFPACK.

Before describing the book in detail, it is sensible to first describe the DIFFPACK environment. Starting in 1990, Are Bruaset and the author, Hans Langtanger of the University of Oslo, Norway, began developing an object oriented finite element code, using the computer language `C++`. This was based on previous experience with a `Fortran` finite element code. The initial `C++` classes for DIFFPACK, the basic finite elements, finite differences, matrices, vectors, grids and fields were developed during 1991-1995. The first public release of DIFFPACK was in 1995, and the second release in 1997. DIFFPACK was initially available as free source from the netlib site. In 1997, a company, Numerical Objects was spun off to support DIFFPACK and from 2003 the software has been developed by InuTech and Simula. The software is now commercial, with a cost somewhat comparable to that of MATLAB, with different rates for educational and commercial licences. DIFFPACK is now a well tested sophisticated computing environment which provides extensive support for developing codes for solving partial differential equations using finite element and finite difference methods. Extensive linear algebra modules are also available and provision for state of the art solution techniques, such as adaptive refinement and multigrid solvers are built into the system. The system is used by numerous large organisations and universities, (as advertised on the DIFFPACK web page) and support is provided by an extensive program of training courses. Indeed these courses are based on the book being reviewed. These courses are normally held in Germany at the inuTech headquarters. This company also provides consulting help for DIFFPACK users.

As you can see, DIFFPACK is a complicated system which requires a large investment of time and effort to become familiar with. It is really only for people or groups who want to **develop** large numerical codes for solving substantial real world problems, want the ability to experiment with a wide range of sophisticated solution techniques and want software environment which allows complicated physical processes to be coupled. If this is the case, then the use of DIFFPACK is worth investigating and if chosen, then Langtangen's book becomes almost an indispensable aid to using DIFFPACK efficiently.

If you are interested in a simple environment for solving standard PDE's, then DIFFPACK is not the system to use, and it is worth looking at other computational environments, such as the MATLAB PDE toolbox, or packages such as FlexPDE. Then it would be better to consider computational books which concentrate more on the specific problems at hand. Though I have to admit that it would be hard to find any other computational science book that covers as wide a range of topics as this book.

Let's look at the content of the book. Following the "Teach Numerics along with Diffpack" aim, the book starts with the standard model problems of the heat and wave equations in one space dimension. The reader is led through the process of constructing more and more sophisticated DIFFPACK programs for solving these problems. Starting with simple implementations based on direct array access (ala `Fortran` or `C`) we are led to the development of codes using the built-in DIFFPACK grid and field classes, and then to codes which support sophisticated parameter control, input and output, user interfaces and graphics. Through this process we are introduced to the power of object oriented programming within the context of the DIFFPACK environment. Along the way, the standard numerical methods of finite differences are introduced. The first chapter, (which is 138 pages long) is really an extended tutorial and users manual on the use of DIFFPACK to solve the heat and wave equations. The chapter is liberally sprinkled with useful examples and indeed there is a long section titled "Projects". It is fairly obvious that the book has been developed to support a tutorial and hands on style of presentation.

But the real power of an object oriented computational environment is its ability to help with the development of finite element codes. The "introduction" of the finite element method is extended over the next two chapters. The first provides a self contained mathematical and computational introduction to the finite element method (FEM), together with an in-depth discussion on the development of a one dimensional code. I quite like this as it allows the reader to develop and completely understand a simple FEM code which has all the components of any standard FEM code. But in addition we know that we can "easily" increase the sophistication of our method by using the built-in DIFFPACK facilities, such as data and parameter management, higher order elements, adaptive grid refinement and multilevel solvers. The second FEM chapter concentrates more on the DIFFPACK environment and shows us how to utilise the more sophisticated DIFFPACK facilities.

These first three chapters provide an excellent introduction to the computational solution of partial differential equations, appropriate for graduate students starting a computational research project using DIFFPACK. These chapters could be used as a basis for a computationally oriented numerical PDE course, and indeed as mentioned before, the book is used by inuTech as the basis for their DIFFPACK training courses.

The final four chapters concentrate on more specific problems, the titles for those chapters being: Nonlinear Problems: Solid Mechanics Applications: Fluid Mechanics Applications: and Coupled Problems. The Nonlinear Problems chapter is of general use, while the others are application specific and provide good methods and hints for solving their respective problems. In addition, there are four substantial appendices, ranging over 200 pages, in the areas of Mathematical Topics (Stability and Accuracy), DIFFPACK Topics (e.g. Visualisation), Solution of Sparse Linear System, and Software Support for Linear Systems.

So this book touches on nearly every aspect of the development of finite difference and finite element numerical solution of PDEs. There is mathematical analysis, software engineering, mathematical modelling and numerous topics in between. In a sense this parallels the range of skills that a computational scientist needs to satisfactorily develop robust and efficient computer codes.

This book will be very useful, if not indispensable, for graduate students or researchers, who intend working with Diffpack. It provides an excellent advanced tutorial and users manual for Diffpack, while also providing a wealth of first hand computational experience presented by an excellent computational scientist.

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## A Course in Modern Mathematical Physics Groups, Hilbert Space and Differential Geometry

Peter Szekeres
CUP Cambridge 2005
ISBN 0-521-82960-7

In reviewing this book, one is inevitably confronted with the question, *what is mathematical physics?* And it is fair to respond that mathematical physics means different things to different people. Ludvig Faddeev [1] argues the case that mathematical physics has emerged as a rival to theoretical physics, in the sense that each draws upon different sources of inspiration. In this picture, theoretical physics is driven by experimental data, whereas mathematical physics draws upon the powerful tools and formidable framework of mathematics. Peter Szekeres concurs with this view in writing that "mathematical physicists put the mathematics first, while for theoretical physicists it is the physics which is uppermost". Of course, each approach strives towards the same goal, namely to the ultimate description of the physical world.

The classic texts in mathematical physics, such as Courant & Hilbert [2], Morse & Feshbach [3] and Jeffreys & Jeffreys [4] are essentially books on differential equations and linear algebra. Yet much of modern physics is heavily based on aspects of geometry and topology. Although abundantly clear in fields like general relativity and string theory, in some cases it lies beneath the surface. For example, although there is no sign of it in P.A.M. Dirac's series of algebra-rich papers developing the foundations of quantum mechanics, Dirac insisted that his approach to quantum physics was geometric and not algebraic [5]. It was this geometrical structure that Dirac used to develop his own picture. In *A Course in Modern Mathematical Physics* Peter Szekeres presents a view of mathematical physics with the emphasis on mathematical structures rather than mathematical analysis. It is then shown how much of physics fits within these structures. As examples, the chapter on differentiable forms contains sections on thermodynamics and classical mechanics, while the chapter on connections and curvature contains sections on general relativity and cosmology.

The precise content of the book is best illustrated via the sixteen chapter headings: Sets and structures, Groups, Vector spaces, Linear operators and matrices, Inner product spaces, Algebras, Tensors, Exterior algebra, Special relativity, Topology, Measure theory and integration, Distributions, Hilbert spaces, Quantum mechanics, Differential geometry, Differentiable forms, Integration on manifolds, Connections and curvature, Lie groups and Lie algebras.

The book is based on the author's lecture notes to undergraduates and his vast teaching experience in mathematical physics at the University of Adelaide. As such the book is suitable for advanced undergraduate and beginning graduate students in mathematical and theoretical physics. The assumed level of knowledge is basic calculus and linear algebra, including matrix theory. The style is very readable. Each chapter includes a number of exercises which are intended not to be too difficult. Rather they are designed to test the reader's understanding or complete a proof. On the other hand, the numerous and often multipart problems given at the section ends are more challenging. Each chapter has a short list of references with a general bibliography at the end of the book. At twelve pages long, the index is comprehensive.

The author is to be congratulated for producing an exceptional book which deserves to be another classic text in mathematical physics.

## References


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[5] G. Farmelo, *Dirac's hidden geometry*, Nature **437** (2005) 323.



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## Ramsey theory on the integers


B. Landman and A. Robertson
AMS Providence 2003
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The authors begin by defining Ramsey theory as "the study of the preservation of properties under set partitions". This rather unsatisfying definition fails to convey the essence of the subject. No matter, for it is often hard to define a mathematical discipline (or indeed, to define *the* mathematical discipline, as the current debate in these pages attests). A typical Ramsey theory result says that at least one of a small set of substructures is sure to occur inside any sufficiently large structure. The structures might be graphs (as in Ramsey's original theorem), groups, vector spaces or, as in this book, sets of natural numbers.

By narrowing their focus to the integers the authors (a) keep the material coherent, (b) avoid needing a lot of background, (c) allow themselves to give a comprehensive treatment in 300 odd pages and yet (d) manage to take the reader right to the coalface of mathematical research. This seems a worthy combination of achievements.

The backbone of the book is van der Waerden's classic theorem that there exists an integer $w = w(k, r)$ such that the integers $\{1, 2, \ldots, w\}$ cannot be coloured with $r$ colours without creating a monochromatic $k$-term arithmetic progression. In other words, the set $\{1, 2, \ldots, w\}$ cannot be partitioned into $r$ subsets without one of those subsets containing a $k$-term arithmetic progression. Chapters 2 to 7 deal with a (surprising, but at the same time almost wearisome) host of variations on this theme: working modulo $m$, arithmetic progressions which are predominantly one colour, monochromatic sequences where each term is some polynomial function of its predecessor, etc, etc.

Chapter 8 deals with Schur's theorem that there exists an integer $s = s(r)$ such that any $r$-colouring of $\{1, 2, \ldots, s\}$ contains a monochromatic solution to $x + y = z$. Then chapter 9 covers Rado's generalisation which gives conditions under which there must be a monochromatic solution to a system of linear equations.

The final chapter deals with assorted further topics such as double-free sets, diffsequences, zero sums and even a superficial glance at Thue-Morse sequences.

As is probably inevitable in a full length book there are some bloopers in the text.

For example, the descending wave constructed in Theorem 3.21 is actually an ascending wave. Also, Theorem 7.17 is vacuous since $j(n) \geq O(n^{1/4})$ includes the possibility that $j(n)$ is bounded... it certainly does NOT imply that $j(n)$ grows as fast as $cn^{1/4}$ for some constant $c$. (The authors fall for this trap several times.) However such errors are rare enough that they do not ruin the book.

The writing style is clear, friendly and accessible. Good explanations are given for the context of each result, together with clear examples. The structure and flow of material is logical and well thought out. The authors recognise the importance of good indexing and a glossary of notation, although the latter is missing a few things (such as asymptotic notation and the use of $\sigma^c$ to denote a string of $c$ copies of a symbol $\sigma$).

A feature of the book is that each chapter ends with a set of exercises and then a (surprisingly long) list of enticing research problems. It seems that there are still many basic questions unanswered, making this book an excellent starting point for a research career. It will also prove a very useful reference for established researchers, although I suspect that a number of the open problems will not be open for long, partly as a result of the spur this book provides. Others have the look of problems which will remain open for many a year.

There are a number of good books covering Ramsey theory but to my knowledge this is the first comprehensive treatment of Ramsey theory for integers. As such it definitely fills a need for a reference in this interesting and active niche of mathematics. The book is accessible with a minimum of background knowledge, meaning that it could be used as the basis for a reading course. As mentioned, it would also be an ideal instrument for introducing a student to research since it is a large reservoir of unsolved problems of (it seems) varying difficulties. One of those problems has already caught my eye, so I'm going to sign off on this review now so that I can work on it...

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## Concrete Abstract Algebra: from Numbers to Gröbner Bases

Niels Lauritzen
CUP Cambridge 2003
ISBN-10: 0521826799

This slim and well written introduction to "abstract algebra" covers an impressive amount of ground, hitting some of the highlights of "elementary" mathematics such as Gauss' law of quadratic reciprocity and Hilbert's basis theorem. It is the best book on abstract algebra that I have come across recently—and it is the only one that I have actually bought!

The book aims to motivate and inspire the reader through "a lot of non–trivial and fun topics". Lauritzen has thought carefully about how to introduce each of these topics and he spices the book with many well chosen examples as well as real applications, primarily from cryptography. The arguments in the book are concise and easy to follow and they are written in such a way so to convey the beauty and elegance of the mathematics. Each chapter comes with an abundance of exercises to keep the reader honest.

The book opens with a chapter on elementary number theory, starting with mathematical induction, the Euclidean algorithm, factorization and then builds slowly up to quadratic residues. These topics are motivated by an entertaining treatment of RSA encryption. The following chapters of the book introduce, in turn, group theory (up to the Sylow theorems), rings (homomorphisms, unique factorization, Euclidean domains), polynomial rings (primitive roots, public key encryption, ideals, finite fields, quadratic reciprocity). The final chapter of the book uses Gröbner bases to give a constructive proof of Hilbert's basis theorem.

Up until now my favourite introductory book on algebra has been Herstein's classic *Topics on algebra*, which is now out of print. Although not as comprehensive as Herstein, the book under review is a worthy "modern" successor. I would be happy to use it as an undergraduate textbook for advanced second year or for third year students.

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Computational Partial Differential Equations Numerical Methods and Diffpack Programming H.P. Langtangen Springer-Verlag, 1999 Purchase from: Amazon.com. The target audience of this book is students and researchers in computational sciences who need to develop computer codes for solving partial differential equations.Â  Spectral and High Order Methods for Partial Differential Equations : Proceedings of the Icosahom 89 Conference Villa Olmo, Como (Italy 26-29 June 19) Claudio Canuto, Alfio Quarteroni (Editor) Elsevier Science Ltd, 1990 Purchase from: Amazon.com. Spectral Methods in Fluid Dynamics Springer Series in Computational Physics C. Canuto, M. Y. Hussaini, A. Quarteroni and T. A. Zang Springer-Verlag, 1988 Purchase from: Amazon.com. Introductory Programming. Life Sciences. Transportation.Â  Home Â» Courses Â» Aeronautics and Astronautics Â» Numerical Methods for Partial Differential Equations (SMA 5212) Â» Lecture Notes. Lecture Notes. Course Home.Â  Numerical Methods for PDEs, Integral Equation Methods, Lecture 1: Discretization of Boundary Integral Equations (PDF - 1.0 MB). (PDF). Numerical Methods for PDEs, Integral Equation Methods, Lecture 2: Numerical Quadrature (PDF). (PDF). Numerical Methods for PDEs, Integral Equation Methods, Lecture 3: Discretization Convergence Theory (PDF). (PDF). Numerical Methods for PDEs, Integral Equation Methods, Lecture 4: Formulating Boundary Integral Equations (PDF). (PDF).