

The Sudoku Epidemic

By Robin Wilson

The sudoku craze has become an epidemic. Originating in the US and Japan, and hardly known a year ago, it has since swept the world, infecting much of Europe, Australia and Asia before returning to the US. So what is sudoku, where did it come from, and how does one solve it?

What is sudoku?

Sudoku (or su doku) is a number puzzle, usually consisting of a 9 x 9 grid divided in nine 3 x 3 boxes, into which numbers already appear in a few cells. Here's an example (Puzzle 1):

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | | | | | 7 | | 1 | |
| | 7 | 4 | | | 2 | | 6 | 5 |
| | 1 | | 8 | 9 | | 4 | | |
| 2 | | | | 8 | | 5 | 9 | |
| 1 | | | 2 | | 3 | | | 6 |
| | 4 | 5 | | 1 | | | | 2 |
| | | 7 | | 4 | 1 | | 2 | |
| 3 | 2 | | 9 | | | 8 | 5 | |
| | 6 | | 5 | | | | | 7 |

Puzzle 1

The object of the puzzle is to complete all the remaining cells with the numbers from 1 to 9, so that:

- each row contains all the numbers from 1 to 9
- each column contains all the numbers from 1 to 9
- each 3 x 3 box contains all the numbers from 1 to 9

The completed pattern is thus a 9 x 9 Latin square with an extra condition on the 3 x 3 boxes.

A well-constructed sudoku puzzle has only one solution which one can find by proceeding in a logical step-by-step manner. The solution to Puzzle 1 is given on page 26, but you may wish to try it for yourself first.

Because of the simplicity of its rules, sudoku has caught the imagination of young and old alike. Its popularity arises partly

from the fact that the puzzles can vary in difficulty from the easy to the extremely hard, depending on how many numbers are given and where they're placed, so solvers can choose a puzzle to match their level of ability.

Why 'sudoku'?

Despite its name, sudoku isn't of Japanese origin. Claims have been made that the puzzle can be traced back to Euler in the 18th century, but these claims are incorrect – Euler certainly worked on Latin squares, but never considered the extra condition on the 3 x 3 boxes. In fact, the first known sudoku puzzle appeared in a New York puzzle magazine in 1979, under the name of *Number place*.

The puzzles didn't reach Japan until the mid-1980s. The puzzle company Nikoli saw an American number place problem and introduced it to the readers of their puzzle magazine in 1984 under the name *Sunji wa dokushin ni kagiru*, shortened to *sudoku* – meaning 'single number' because just one number appears in each cell. They soon became highly popular, and currently some 15 Japanese sudoku magazines and 30 books are published monthly, catering to half a million readers.

In 1997, Wayne Gould, a retired High Court Judge living in Hong Kong, discovered sudoku in a Tokyo puzzle book. Fascinated, he spent the next few years designing computer programs to produce sudoku puzzles of varying difficulty which he then provided for newspapers around the world.

In 2004, Gould sent one to *The Times* in London, who published their first puzzle in November – his puzzles still appear, in four levels of difficulty (easy, mild, difficult, and fiendish). Other newspapers followed suit, and by Easter 2005 a huge cottage industry had developed, with most British newspapers including daily sudoku puzzles and rivalling each other to offer prizes for solutions to puzzles of ever-increasing complexity and originality. These now include 16 x 16 sudoku grids, grids of five linked 9 x 9 sudoku puzzles (none completable on its own), and *The Times* 'killer puzzles' in which the numbers in certain subsets of cells have specified sums. There is even a three-dimensional sudoku puzzle in the form of a 9 x 9 x 9 cube.

In May 2005 the sudoku craze suddenly spread around the world. Within a month sudoku puzzles had appeared in newspapers in Australia, Canada, Israel, Eastern Europe, and India, and they have recently returned to haunt the United States, now appearing in many dozens of American newspapers.

Further historical notes can be found in [5].

Solving a sudoku puzzle

Suppose that you are given the sudoku puzzle on the right to complete (Puzzle 2).

We'll call the rows A, B, \dots, I , the columns a, b, \dots, i , and the 3×3 boxes ①, ②, \dots , ⑨, proceeding horizontally from top-left (box ①) to bottom-right (box ⑨). We denote cells by their row and column letters – for example, Cf is the cell in row C and column f (containing 9).

There are several techniques that you can use to solve sudoku puzzles, such as: *scanning rows and columns*, *filling in the gaps*, and *filling in single cells*; further details can be found in [3]:

Scanning rows and columns

Look first at columns d, e, f of Puzzle 2. In these columns, each number from 1 to 9 must appear three times in total, once in each column and once in each 3×3 box; for example, the number 3 appears in cells Bf (box ② and column f), Ee (box ⑤ and column e), and Gd (box ⑧ and column d). It follows that:

- the third appearance of 9 must be in cell Id
- the third appearance of 5 must be in cell Ad

In the same way, we notice that:

- in rows G, H, I , 5 must be in cell Ii (since 5 already appears in column h)
- in columns g, h, i , 6 must be in cell Hi (since we have just put 5 in cell Ii)
- in box ⑤, 6 cannot be in rows D or E , so must be in row F , in cell Fd ; similarly, 7 must be in cell Ff

More subtle is the following:

- in box ⑨, 8 is in column g ; in box ⑥, 8 cannot be in column g or row E , so must be in column i , in cell Di or Fi ; thus, in box ③, 8 must be in column h , in cell Ah

Filling in the gaps

When rows, columns and boxes are almost complete, we can sometimes fill in the missing entries; for example:

- in column d , only 1 is now missing – so put 1 in cell Dd
- in box ⑤, only 4 is now missing – so put 4 in cell Df
- in column f , 1 and 2 are missing, but we cannot yet decide where to put them

Filling in single cells

| | a | b | c | d | e | f | g | h | i |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A | | | 4 | | | | 6 | | |
| B | | 2 | | 4 | | 3 | | 1 | |
| C | 1 | | | 8 | | 9 | | | 4 |
| D | | 7 | | | 5 | | | 6 | |
| E | | | | 2 | 3 | 8 | | | |
| F | | 1 | | | 9 | | | 5 | |
| G | 5 | | | 3 | | 6 | | | 7 |
| H | | 9 | | 7 | | 5 | | 2 | |
| I | | | 7 | | | | 8 | | |

Puzzle 2

Look at cell Bi . It cannot contain 1, 2, 3 or 4 (already in row B), 5, 6 or 7 (in column i) or 8 (in box ③), so must contain 9; we can then put 9 in cell Aa .

Similarly, cell Ab cannot contain 4, 5, 6 or 8 (already in row A), or 2, 7, 1, 9 (in column b), so must contain 3.

Note also that in column b , the only place that we can put 8 is in cell Gb ; we can then put 8 in cell He .

Using small numbers

We could continue like this, but another approach is to write in each square the possible numbers that can be inserted. For our puzzle the current position is:

| | a | b | c | d | e | f | g | h | i |
|-----|------|-----|------|-----|-----|-----|------|-----|-----|
| A | 9 | 3 | 4 | 5 | 127 | 12 | 6 | 8 | 2 |
| B | 678 | 2 | 568 | 4 | 67 | 3 | 57 | 1 | 9 |
| C | 1 | 56 | 56 | 8 | 267 | 9 | 2357 | 37 | 4 |
| D | 238 | 7 | 2389 | 1 | 5 | 4 | 239 | 6 | 238 |
| E | 46 | 456 | 569 | 2 | 3 | 8 | 1479 | 479 | 1 |
| F | 2348 | 1 | 238 | 6 | 9 | 7 | 234 | 5 | 238 |
| G | 5 | 8 | 12 | 3 | 124 | 6 | 149 | 49 | 7 |
| H | 34 | 9 | 13 | 7 | 8 | 5 | 134 | 2 | 6 |
| I | 2346 | 46 | 7 | 9 | 124 | 12 | 8 | 349 | 5 |

Puzzle 2 (continued)

Note that:

- in cells A_i and E_i , only one number can be inserted, thereby enabling us to fill in further squares
- in box ①, the cells C_b and C_c must contain 5 and 6 (in some order), so cell B_c must contain 8 and cell B_a must then contain 7

Continuing in this way, you should be able to complete the puzzle – the solution is on page 26.

Some mathematics

A number of interesting mathematical problems arise in connection with sudoku. The number of possible 9×9 Latin squares is

$$5,524,751,496,156,892,842,531,225,600;$$

how many of these are valid sudoku grids? In June 2005, this number was found by Felgenhauer and Jarvis [1] to be

$$6,670,903,752,021,072,936,960.$$

Later, the number of ‘essentially different’ grids (once relabelling of the nine symbols and obvious row and column permutations have been eliminated) was found by Russell and Jarvis [2] to be 5,472,730,538.

It is also natural to ask for the maximum and minimum number of filled-in cells for a sudoku puzzle to have a unique solution. The maximum number is 78 – if from the solution of Puzzle 2 the 1s and 9s in positions A_a, A_f, C_a and C_f are erased, leaving 77 filled-in cells, the puzzle cannot be uniquely completed. The minimum number seems to be 17. There are several known examples with 17 filled-in cells which can be completed uniquely, but no-one has found one with just 16 filled-in cells, or proved that none can exist. Further information can be found in [5].

Summing up

It only remains to wish you all the best in your sudoku explorations. Solving these puzzles can be very frustrating at times – but it can also be enormous fun! We conclude with a puzzle (Puzzle 3) containing nine letters – if you complete it correctly, a hidden mathematical word will appear in one of the rows or columns; the solution is given on page 26. Further word puzzles can be found in [4].

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | H | T | | G | |
| T | | | | | | | | |
| | | R | M | | | H | | |
| H | | | L | | I | O | | |
| G | | | | | | | | A |
| | | L | T | | R | | | M |
| | | O | | | H | I | | |
| | | | | | | | | L |
| | M | | O | R | | | | |

Puzzle 3

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References

1. B. Felgenhauer and F. Jarvis, Enumerating possible Sudoku grids, <http://www.shef.ac.uk/~pm1afj/sudoku/sudoku.pdf>.
2. E. Russell and F. Jarvis, There are 5472730538 essentially different Sudoku grids ..., <http://www.shef.ac.uk/~pm1afj/sudoku/sudgroup.html>.
3. R. Wilson, *How to Solve sudoku: A step-by-step guide*, Infinite ideas, 2005.
4. R. Wilson, *Hidden Words sudoku*, Infinite ideas, 2005.
5. A useful website that gives information about all aspects of sudoku and is updated regularly is <http://en.wikipedia.org/wiki/Sudoku>.

Play the ever entertaining game of Sudoku online on TheSudoku.com - fun and packed with helpful features. Note: we're still tuning up the gameplay so if you feel something's wrong or would like some feature added, just let us know. Sudoku puzzles (Su Doku). My database includes 6913752 puzzles for the moment and I'm constantly generating more. Rules: Fill in the grid so that every row, every column, and every 3x3 box contains the digits 1 through 9. There is only 1 solution for each puzzle. For more information check out Wikipedia. 106486499 puzzles downloaded. Navigate the grid with the arrow keys or your mouse; clear entries with Delete or Backspace. Once a number is in the grid nine times, it will lighten. You can disable this in Settings. Use Fill Mode (the big 1 button) to enter numbers you are confident about. Use Candidate Mode (the nine-square button) to add or remove multiple possibilities for a square. In Fill Mode, you can hold down Shift or Alt to temporarily enter Candidate Mode.