On Endogenizing Long-Run Growth

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ABSTRACT: This assessment of recent theoretical work on endogenous growth identifies three different engines of long-run growth: (i) the asymptotic average product of capital is positive; (ii) labor productivity increases as an external effect of capital accumulation; (iii) there are feedback effects on the cost of accumulating knowledge or innovating. A general model encompassing all three is considered, and then used to review different proposed determinants of long-run growth rates. The contribution of endogenous growth theory has been to create a framework in which to explain why economic institutions and policies can have long-run effects on growth rates.

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I. Introduction

Progress in economic science often takes the form of explaining what was previously inexplicable. That is, variables which had earlier been treated as exogenous become endogenized. Their values become determined, at least in principle, within an economic model.

Much early economic analysis restricted itself to the short run, in which the stock of capital equipment and workers' skills were treated as exogenously fixed by history. Progress toward a theory of growth came from adding a theory of investment which would determine endogenous changes in capital stocks, and a comparable theory of skill or "human capital" acquisition for workers. In the medium term, rates of economic growth are obviously affected by a vast range of factors, including saving behavior, measures to stimulate private investment, provision of public goods whose benefits extend into the future, and policies affecting banking, finance, international trade, health, education, etc. Hence there is no shortage of ways to explain why growth occurs in the medium term, or how fast such growth can be. Accordingly, there is no shortage of medium term endogenous growth models.

In the long run, however, the situation is quite different. For thirty years, overlapping generations of students have been taught by their predecessors that long run growth rates, if they are not zero, are determined by exogenous, non-economic factors such as population growth and the rate of scientific discovery. Indeed, as is well known, the orthodox neoclassical model of growth with no exogenous technological progress implies that the economy will reach a steady state in the long run with zero growth in income per head. Thus, according to this model, measures to promote growth can only enhance short- and medium-run growth rates, as well as long-run *levels* of consumption and output; it became the prevailing view that long-run growth rates could not really be affected at all by economic policy. In this sense, the engine of growth was seen as entirely exogenous. The long-run rate of growth was endogenous only in a trivial sense, since it was entirely determined by exogenous technical progress or population growth.

Recently, a serious challenge to this orthodoxy has been mounted. Actually, related ideas could already be found in earlier work on growth theory and development by Stigler (1951), Haavelmo (1954, 1956), Tobin (1955), Solow (1956), Myrdal (1957), Schultz (1961, pp. 5–6; 1962) and Arrow (1962), amongst those we know now were destined to win Nobel prizes later on.[1] Yet it took the recent work of Romer (1986, 1987, 1990), Krugman (1987), Prescott and Boyd (1987a, b), Lucas (1988) and others to set off a revolution. These authors developed various models in which long-run growth rates could be determined by the same kind of factors that had previously been regarded as affecting only short- or medium-term growth. In the process, they created what has come to be called "endogenous growth theory" in which the engine of growth has become part of the model itself: it results from the maximizing actions of individual economic agents. This is like the transformation from a carriage with an exogenous horse to a horseless carriage or automobile with its own built-in source of motive power.

This paper surveys these recent developments in the theory of economic growth. In our view, three major themes have emerged. The first concerns how to modify the orthodox neoclassical model so that endogenous long-run growth in income per capita becomes possible. The classic works in the new growth literature, including Romer (1986, 1990), Lucas (1988), Jones and Manuelli (1990), Rebelo (1991), and Grossman and Helpman (1991a), all emphasize this theme.

Second, there has been considerable analysis of how the growth rate can be affected by variables such as public goods, finance, trade, taxes, demographic parameters, income distribution and social norms, all of which played no role in the traditional neoclassical theory. Recently, many papers have appeared that take an endogenous growth model, extend it to incorporate at least one of these extra variables, and then perform a comparative static analysis of how these variables influence long-run growth.

Finally, the third major theme concerns how growth can be propagated across countries that are linked through flows of goods, capital and ideas. For instance, if technological innovation is the

[1] See Gunnar Myrdal's "principle of circular and cumulative causation." One distinguishing feature of the new theory is its reliance on external economies of scale. This is an idea that had been explored by Alfred Marshall, Allyn Young, Tibor Scitovsky, Nicholas Kaldor, Albert Hirschman, Nathan Rosenberg, and many others besides those whose works we mention explicitly in the paper.

engine of growth, then it would seem that the growth potential of countries that have not reached the technological frontier is larger than for the innovating countries. These issues, however, are far from being well understood.

As can readily be appreciated, the new growth literature is already enormous. Indeed, several other symposia and surveys have already appeared, or will do so shortly.[2] For this reason, and due to limitations of space and time, our survey is restricted in many ways. First, we will not review systematically the literature on the third major issue concerning how growth is propagated internationally, though our discussion of the relation between trade and growth is of some relevance. In some sense, this means that we leave aside the development issues that are most pertinent to relatively poor countries. Second, given our own comparative advantage, we focus on the theoretical part of the literature, virtually excluding all mention of its very important empirical side. Finally, the survey attempts to be self-contained and to derive most of its results analytically; as a consequence, the number of models and papers which can be reviewed in any detail is quite small and so many important contributions will not be mentioned at all. We believe that this is a price worth paying, however, since it permits us to be more rigorous and didactic in explaining what we think is most important.[3]

In what follows, section 2 begins by reviewing the orthodox neoclassical growth model. Then sections 3, 4 and 5 are devoted to the first major theme. They present three different and fundamental departures from the orthodox model that all allow endogenous long-run growth of income per capita. These three sections cover the major basic features of the new growth theory.

Next, section 6 develops a more general model that embodies all these three different approaches. It also derives some general results about how several different exogenous variables affect the growth rate. This allows a better understanding of the essential aspects of the three different approaches to endogenous growth that were outlined in sections 3–5. Thereafter, section

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[2] We have noticed surveys or prominent lectures by Lucas (1988, 1993), Romer (1991), Stern (1991), Sala-i-Martin (1990a, b), Helpman (1992), Hahn (1992), Solow (1993), and other symposia in a supplementary issue of the *Journal of Political Economy*, 1990, in the May 1991 issue of the *Quarterly Journal Economics*, in the Winter 1992 issue of the *Oxford Review of Economic Policy*, and in the December 1992 issue of the *Journal of Economic Theory*. There have even been newspaper articles in the *Economist* (4th January 1992, p. 17) and by Brittan (1993).

[3] For a much more extensive survey, see Gans (1989).

7 briefly reviews the main literature on specific determinants of growth, linking its predictions to the results of section 6 wherever possible. The final section 8 is a concluding assessment.

II. The Orthodox Neoclassical Model of Purely Exogeneous Growth

Neoclassical growth theory has many significant predecessors. These include the “classical” growth theories of Smith, Ricardo, Malthus, Marx, etc. (see Harris, 1987). There are also the “Keynesian” growth models of Harrod (1939) and Domar (1946).[4] Tobin’s (1955) paper was a significant addition to this line of work, as it made the growth rate depend on the elasticity of labor supply, amongst other variables. Modern endogenous growth theory, however, takes as its point of departure the very similar orthodox neoclassical equilibrium growth models of Solow (1956) and Swan (1956).[5] In their purest form, these predict that the capital–labor ratio converges to some long-run equilibrium value, as do the real wage, the rate of return to capital, and the level of income per capita. These long-run equilibrium values do depend on the saving rate. But the long-run rate of growth of output is in a sense exogenous, being equal to the rate of population growth. In the long run, consumption per capita converges to a stationary equilibrium value. Policies to enhance growth can only influence the short term.

Let L denote the amount of available labor. In many growth models it is assumed that L grows at a fixed exponential exogenous rate. Yet, as biologist Paul Ehrlich (1968) for one has pointed out so strikingly, were population growth to continue indefinitely without the physical size of our heirs shrinking to zero, then ultimately the human race could not be contained within any sphere expanding more slowly than the speed of light! In any case, since the question is whether income per head can grow, this exogenous population growth rate will not materially affect most of our results.[6] So for simplicity most of the rest of the paper will simply assume it is zero. Moreover we will often refer to L as a *non-reproducible resource* because it has an exogenous supply that is

[4] This is actually somewhat preceded by the work of Fel’dman (1928), as Domar (1957) eventually acknowledged.
[5] See also the collections of articles by Newman (1968), Stiglitz and Uzawa (1969), and Sen (1970), as well as the thorough survey by Hahn and Matthews (1964).
[6] In some endogenous growth models such as Romer (1990), population growth may lead to an explosive rate of output growth. See Jones (1993).

fixed in the long-run. Indeed, a somewhat similar model, with L representing the rate of depletion of an exhaustible resource rather than labor supply, was used by writers such as Dasgupta and Heal (1974), Solow (1974), and Stiglitz (1974). In their analysis of growth with exhaustible resources, the value of this L had to converge to zero in the limit. It turns out that the possibilities for long-run growth are much the same as when L is a fixed quantity of labor.

To see why income per capita cannot grow for ever in the orthodox neoclassical growth model, note first that it postulates a constant returns to scale (CRS) production function. Specifically, when output Y is measured net of capital depreciation, it is given by a function

$$Y = F(K, L) \tag{1}$$

of L and of a single capital good K. In this one sector growth model, net investment is given by

$$\dot{K} = F(K, L) - C \tag{2}$$

where C denotes consumption and a dot over any variable is used to denote its time derivative. Differentiating (1) with respect to time gives

$$\dot{Y} = F_1(K, L)\,\dot{K} \tag{3}$$

where F_i will always denote the partial derivative of F with respect to the i-th argument. Suppose that net investment $\dot{K}$ is a positive constant proportion s of net output Y. Hence, using (3), and indicating the proportional growth rate of any variable by a hat, output grows at rate

$$\hat{Y} = \dot{Y}/Y = sF_1(K, L) \tag{4}$$

Section 6 below will present a growth model in which s is determined by the intertemporal maximization of infinitely lived representative agents. For the purposes of this section, however, the original and only slightly cruder assumption (due to Harrod, Domar, Solow, and Swan) of a fixed s for all time is sufficient to establish the main points.

At this stage, in order to ensure existence of a steady state with a positive level of output, neoclassical growth theory has usually invoked the Inada (1963) conditions. There are two of these. The first *lower* condition requires that $F_1(K, L) \to \infty$ as $K \to 0$. More interesting for us, however, is the second *upper* condition requiring that

$$F_1(K, L) \to 0 \text{ as } K \to \infty \tag{5}$$

Given this upper condition, (4) implies that the rate of growth of output tends to zero in the limit. Provided that $s \leq 1$, this is true no matter what the value of s may be. Indeed, it remains true even when s(t) varies with t, but with $s(t) \leq 1$ for all t.

The next three sections consider three different modifications of the neoclassical model that make long-run growth possible. Of these, the first involves simply dropping the crucial upper Inada condition (5). The second modification asserts that a by-product of capital accumulation is an increase in the productivity of labor or of other non-reproducible factors. Reasons given for this have included: (i) learning by doing; (ii) government provision of public services financed out of taxation; (iii) capital accumulation allowing a deeper division of labor. The third modification also relies on an accumulation process having a by-product, but emphasizes how increasing knowledge or "human capital" makes innovation and/or education less costly.

III. Inessential non-reproducible factors: the AK model

The simplest departure from the previous model with no technical change is to relax the upper Inada condition (5), so that the marginal productivity of capital does not tend to zero as the capital–labor ratio goes to infinity. This assumption means that only reproducible factors are "essential" in the sense that, as their input levels approach zero, so the marginal products of other factors also converge to zero.

In a prominent example of this approach, Jones and Manuelli (1990) explicitly assume that

$$F_1(K, L) \to \mu > 0 \text{ as } K \to \infty \tag{6}$$

In this case, (4) and (6) together imply that the growth rate of Y is bounded away from zero and becomes $s\mu > 0$ in the limit. The long-run rate of output growth is endogenous, being determined by the rate of capital accumulation.

A notable example that satisfies (6) is the constant elasticity of substitution (CES) function

$$F(K, L) = [\alpha K^{\rho} + (1-\alpha)L^{\rho}]^{1/\rho}, \quad 0 < \rho \leq 1 \tag{7}$$

Of course, (7) takes on the Cobb-Douglas form $F(K,L) = K^{\alpha}L^{1-\alpha}$ when $\rho = 0$. As is well known, the condition $\rho \leq 1$ is necessary for F to be concave. Moreover, $\rho > 0$ implies that, as $K \to \infty$, so

$$F_1(K, L) = \alpha K^{\rho-1}[\alpha K^{\rho} + (1\text{-}\alpha)L^{\rho}]^{(1/\rho)-1} = \alpha[\alpha + (1\text{-}\alpha)(L/K)^{\rho}]^{(1/\rho)-1} \to \alpha^{1/\rho} > 0 \qquad (8)$$

On the other hand, it is easily checked that $F_1(K, L) \to 0$ as $K \to \infty$ when $\rho \leq 0$. In fact $\rho > 0$ ensures that the elasticity of substitution $\sigma = (1-\rho)^{-1}$ exceeds 1: this is necessary for the share of capital in output to be bounded away from zero as the capital–labor ratio goes to infinity. Actually, Solow (1956, p. 77) himself considers this possibility (especially when $\rho = 1/2$), and describes the economy as "highly productive" in this case.

Another simple example satisfying (6) is

$$F(K, L) = \mu K + BK^{\alpha}L^{1-\alpha}, \quad 0 < \alpha < 1, \ \mu > 0 \qquad (9)$$

Rebelo (1991), as well as Barro and Sala-i-Martin (1992), postulate a model in which the production function is as in (9), but with $B = 0$, so that only reproducible resources are used as inputs. This is often referred to as the "AK model" because the production function can be expressed as $F(K, L) = AK$. Of course, it gives the same asymptotic properties as if $B > 0$ in (9), since the second term becomes relatively unimportant in the limit as $K \to \infty$.

Rebelo also shows that even when non-reproducible resources are essential in production, as they are in the Cobb-Douglas case, growth is still possible as long as there is at least one capital good whose production uses only reproducible resources. This can be seen in a two sector growth model, with output C of the consumption good being produced according to a Cobb-Douglas production function using capital K_C and labor L, so that

$$C = K_C^{\alpha}L^{1-\alpha}, \quad 0 < \alpha < 1 \qquad (10)$$

while the investment sector uses only capital and exhibits CRS, so that

$$\dot{K} = aK_I, \ a > 0 \qquad (11)$$

where $K_C + K_I = K$. To see how growth is then possible in the long run, just note that if a constant fraction ϕ of the capital stock goes to produce investment goods, then $K_I = \phi K$, while $K_C = (1\text{-}\phi)K$. Hence (10), (11) and ϕ jointly determine the constant growth rate

$$\hat{C} = \alpha \hat{K}_C = \alpha \hat{K} = \alpha a \phi \tag{12}$$

The same principle lies behind the Lucas (1988) model of growth through skill acquisition or human capital accumulation. In his model, which is based in turn on work originally carried out by Uzawa (1965), skill augments the efficiency of labor. Moreover, skill is passed from generation to overlapping generation. In addition, externalities allow individuals to accumulate skill more easily when their parents' human capital stock is higher. Also, individuals spend fractions u of their time producing output and 1-u increasing their human capital H. So the model is described by

$$\dot{K} + C = Y = F(K, uLH) \tag{13}$$

$$\dot{H} = \xi(1-u)LH \tag{14}$$

where ξ is a positive parameter that indicates how effective is time spent learning.[7] For any $u < 1$, long run balanced growth is obviously possible at a rate g given by

$$g = \hat{C} = \hat{K} = \hat{H} = \xi(1-u)L \tag{15}$$

provided that u and the savings ratio $s = \dot{K} / Y$ can both be chosen to satisfy

$$\dot{K} = gK = sY = sF(K, uLH) \tag{16}$$

for all time. Thus, according to (15), the long-run rate of growth is determined by: (i) the exogenous supply of labor L; (ii) the proportion 1-u of labor effort that goes into acquiring skills; (iii) the rate of learning parameter ξ.

IV. Increasing labor productivity as an external effect of capital accumulation

The second modification of the neoclassical growth model is most clearly expressed by introducing a continuum of identical representative agents. Each agent's output is assumed to be given by a CRS production function

$$y = F(k, E\,l) \tag{17}$$

[7] Lucas also assumes that the productivity of any worker's own human capital increases with the social average level of human capital. Concretely, he assumes a production function of the form $y = F(K, uLH)\, g(H)$, where $g' > 0$. This is done to make the results of the model appear more realistic. It is not necessary for the economy to be able to grow in the long run. Another difference from our formulation is that Lucas also has another reproducible asset — physical capital. Again, this is not the main insight of his model and so we omit it from this discussion. Both physical and human capital do appear, however, in the general model we set out in section VI.

where, for that agent, k is the stock of available capital and l is the input of labor. Here E represents the efficiency of labor, which is common to all agents. In fact it is assumed that

$$E = A(K) \tag{18}$$

where $A(K)$ is an increasing function of the aggregate capital stock K. That is, labor (or any other non-reproducible resource) becomes more productive as a direct external effect of capital accumulation (cf. Swan 1956, p. 338). Aggregating over all identical agents implies that

$$Y = F(K, A(K)L) \tag{19}$$

Three main justifications for this assumption will be discussed in the ensuing subsections. First, however, we investigate the conditions under which this general one sector model allows long-run growth. When investment is a constant fraction s of output, (19) implies that the growth rate of aggregate output is

$$\hat{Y} = \dot{Y} / Y = (F_1 + F_2\, A'(K)\, L)\ \dot{K} / Y = sF_1(K, A(K)L) + sF_2(K, A(K)L)\, A'(K)\, L \tag{20}$$

Suppose that, because of the usual upper Inada condition (5), the term sF_1 in (20) tends to zero as $K \to \infty$. Even so, there can still be long run growth provided that the term $sF_2A'(K)L$ is bounded away from zero. Because of CRS, the partial derivative F_2 is homogeneous of degree zero, and so

$$F_2(K, A(K)L)\, A'(K)\, L = F_2(1, A(K)L/K)\, A'(K)\, L \tag{21}$$

A sufficient condition for the expression in (21) to be bounded away from zero is that $A'(K)L$ converges to $b > 0$ as $K \to \infty$. For then $A(K)L / K \to b$ also, and so $F_2(1, A(K)L / K)\, A'(K)\, L$ tends to $F_2(1, b)\, b$. Finally, therefore, the asymptotic growth rate of output is

$$\hat{Y} = sF_2(1, b)\, b > 0 \tag{22}$$

which depends once again on the asymptotic rate of saving s, as well as on the parameter b.

IV.1 *Learning by Doing and Social Knowledge*

One justification for the technological possibilities described by (19) was originally offered by Haavelmo (1954; 1956, pp. 36-39) in a growth model with a single aggregate linear production function. He postulated that aggregate learning by doing results from the investment process, so that the present level of education or knowledge of the workforce is a function of the capital stock. This was before Arrow (1962) gave prominence to the same idea in a more plausible growth model with vintage capital and fixed coefficients, along the lines that had recently been pioneered by Johansen (1959), Salter (1960) and Solow (1960).[8]

Thereafter, following Levhari (1966), Sheshinski (1967) reformulated Arrow's model to exclude vintage effects, so that it became described by (19). He then imposed both the upper and lower Inada conditions on the function F — as Arrow had done in an extreme way by assuming fixed coefficients. Sheshinski also assumed that there would be diminishing returns to cumulative investment in the generation of knowledge, in the sense that $A(K)/K \to 0$ as $K \to \infty$. In this case, if σ denotes the rate of growth of L, while $A(K) = K^{\eta}$ with $\eta < 1$, then it is possible for both Y and K to grow at any common rate g satisfying

$$Y_0\, e^{gt} = F(K_0\, e^{gt}, K_0\, L_0\, e^{(g\eta + \sigma)t}) \tag{23}$$

for all t. This makes it clear that constant growth is only possible at a rate satisfying $g = g\,\eta + \sigma$, and so $g = \sigma/(1-\eta)$. Hence the growth rate in income per capita, which is $g - \sigma = \sigma\eta/(1-\eta)$, must be increasing in the population growth rate and zero when there is no population growth.

In his 1986 paper Romer made the accumulation of knowledge result at least in part from private decisions, and not simply remain an unintended consequence of aggregate past investment as it had been in Arrow's interpretation. In effect Romer regarded K, the aggregate stock of knowledge, as a public good from which individual producers could benefit directly. More importantly, Romer considered the possibility that $\eta > 1$. Then $A(K)/K = K^{\eta-1} \to \infty$ as $K \to \infty$ and so, as shown above, there can be positive long-run growth even when population is constant. Indeed, long-run growth is also possible in the possibly more plausible case when $\eta = 1$.

In these models it should be noted that, even though there may be increasing returns to scale in the aggregate, each agent remains with a production function that is concave and has constant returns to scale in the variables under that agent's control. As Chipman (1970) and Romer (1986) point out, perfect competition therefore remains possible, with each producer taking both prices and the aggregate capital stock (or labor productivity) as fixed. In fact, capital accumulation is like

[8] Arrow cites other closely related ideas that had appeared previously, including Verdoorn's (1949, 1980) law and the Swedish "Horndal effect" (dating back to the 1830s) noted by Lundberg (1961) — see also David (1975, ch. 3).

a public good to which each producer contributes privately by investing. Evidently the resulting equilibrium allocation will usually be far from efficient.

IV.2 *Public Services*

Another justification for (18), inspired by Barro (1990), is that E represents public services which enhance labor productivity. For simplicity it is assumed that these services are produced with the same production function as private goods and are financed by a proportional income tax, so that $E = \tau Y$. Then the aggregate form of equation (17) implies that $E = \tau Y = \tau F(K, EL)$. Thus $1 = \tau F(K/E, L)$ because of CRS. Therefore $E = cK$,where c is the positive constant given implicitly by $1 = \tau F(1/c, L)$. This is a special case of (18), of course.

Notice that this formulation makes public services non-rivalrous. This is because an increase in L that leaves E constant would not cause each worker to have fewer public services. However, having these services be public is not a necessary condition for this formulation to allow long-run growth. For if instead public services were to become private to each worker, with G as the total level of such services, then we would have $E = G/L$. It is easy to check that with this modification the results are similar to those above, except that the constant c is given by $1 = \tau F(1/c, 1)$.

IV.3 *Division of Labor*

The final justification to be considered here holds that a deeper division of labor between different specialized tasks increases the productivity of non-reproducible resources. But, to borrow from the memorable title of ch. III of Adam Smith's Wealth of Nations, "the division of labour is limited by the extent of the market."[9] And of course the market can be expected to become more extensive if the capital stock increases. Thus, it is assumed that capital accumulation allows an increase in the productivity of non-reproducible resources — an increase that will actually occur if production is organized efficiently.[10]

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[9] Of course, Stigler (1951) honored Smith by choosing the very same title.
[10] The model presented in this section is adapted from Rodríguez-Clare (1993). It is related to Becker and Murphy (1992), though they give quite different interpretations. The model differs from Romer (1990) who assumes that the fixed requirement to produce a new variety of z involves essential non-reproducible factors and hence externalities in

A simple formulation of this idea postulates that output is produced from capital K_Y in the final output sector and from an intermediate input Z according to a CRS production function

$$Y = F(K_Y, Z) \tag{24}$$

Following Dixit and Stiglitz (1977), assume that Z is produced using quantities z(j) ($0 \le j < \infty$) of a continuum of varieties of an intermediate good according to a strictly concave production function

$$Z = [\int_0^\infty z(j)^\alpha dj]^{1/\alpha}, \quad 0 < \alpha < 1, \tag{25}$$

with constant elasticity of substitution exceeding one. Assume also that each variety of intermediate good z(j) is produced from K(j) units of capital and L(j) units of labor according to

$$z(j) = \begin{cases} 0 & \text{if } K(j) < 1 \\ L(j) & \text{if } K(j) \ge 1 \end{cases} \tag{26}$$

Since producing z(j) involves set-up costs, perfect competition is no longer possible; instead, there will be a monopolistically competitive equilibrium. In fact, this section will only investigate the conditions under which growth is possible. Accordingly we focus on allocations that are instantaneously efficient at each time, rather than going through the more involved derivation of equilibrium that can be found in Rodríguez-Clare (1993). Of course we do not assume intertemporal efficiency, since dynamic externalities make that goal unattainable in equilibrium.

Another implication of set-up costs is that instantaneous efficiency requires not all varieties of the intermediate good to be made available. Since all varieties require the same technology and enter the production of Z symmetrically, it loses no generality to represent the set of available varieties by the interval [0, n] of the real line. Given that the unit of capital is defined as the amount needed to produce each variety of intermediate good, it follows that n must be equal to K_Z, the quantity of capital used in the intermediate goods sector.

In fact the production functions (25) and (26) imply that there are returns from the division of labor in the production of intermediate goods. To see this note that, because of strict concavity ($0<\alpha<1$) and the symmetric way in which different varieties of z appear in (25), instantaneous

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the research sector are need to generate the possibility of long run growth. Yang and Borland (1991) also give the division of labor an essential role in the growth process, but let growth itself be driven by learning by doing.

efficiency requires all firms producing final goods to use the same quantity of all available varieties. Thus $z(j) = z$ for all $j \leq n$. Then (26) implies that $L(j) = z$ for all such j. Hence the total amount of labor devoted to the production of intermediate goods must be $L_Z = nz$. Because $L_Z = L$, it follows that $z(j) = z = L/n$ for all $j \leq n$. Thus (25) implies that

$$Z = [\int_0^n z^\alpha \, dj\,]^{1/\alpha} = n^{1/\alpha} z = n^{1/\alpha} (L/n) = n^\phi L, \text{ where } \phi = (1-\alpha)/\alpha \tag{27}$$

Finally then, the production relation (24) can be written as

$$Y = F(K_Y, n^\phi L) = F(K_Y, K_Z{}^\phi L) \tag{28}$$

which shows how an increase in the measure $n = K_Z$ of varieties available increases the efficiency of labor in producing final goods. This property of the production function in (25) and (26) is commonly referred to as *love of variety for inputs*. It arises because different inputs are imperfect substitutes for each other and so, if fewer varieties of intermediate goods are made available, then the firm will have to use those that are available more intensively. There is a loss of output due to imperfect substitution.[11]

In the special case when $F(K_Y, Z) = K_Y{}^\beta Z^{1-\beta}$ it follows from (28) that $Y = K_Y{}^\beta K_Z{}^{\phi(1-\beta)} L^{1-\beta}$. Then it is easily shown that a instantaneously efficient capital allocation requires $K_Y = K - K_Z = \tau K$, where $\tau \equiv \beta/(\beta+\phi(1-\beta))$.[12] In this case the maximum production of Y given K is

$$Y = F(\tau K, [(1-\tau)K]^\phi L) \tag{29}$$

It is obvious that (29) is a special case of the general model described by equation (19). The corresponding micro version of (29) is $y = F(\tau k, [(1-\tau)K]^\phi\, l\,)$, which is a particular form of (17).

[11] One alternative modelling strategy would be to assume that each firm producing a final good needs different inputs at different times. At any time, a given producer of the final good wants an ideal specialized input; if, however, that input is not available in the market, the firm will buy the "closest" one it finds and transform it, at a cost, into the desired input. The more varieties of the input that are available, the less the firm will have to spend on this transformation and hence the more efficient the firm will be. This alternative model is based on a reinterpretation of Lancaster (1979) proposed by Weitzman (1991).

[12] With a more general production function F, even one that is homothetic, the amount of capital K_Y allocated directly to producing output Y would not be proportional to K, and would also depend on L. Then (19) would need replacing by $Y = F(K, A(K,L)L)$, but with L fixed the difference is trivial. Here we restrict ourselves to the Cobb–Douglas case in order to simplify the exposition.

V. Feedback Effects on the Cost of Learning or Innovation

A third departure from the neoclassical growth model postulates that a by-product of earlier accumulation is a decrease in the cost of later accumulation. For instance, the higher is the "stock of knowledge", the lower is the cost of accumulating more knowledge. Formally

$$Y = F(K, L, H_Y) \tag{30}$$

$$\dot{K} = G(H_I)\, h(K) \tag{31}$$

where L is the size of the labor force, H_Y is the amount of human capital devoted to producing output, and H_I is the level of human capital devoted to the accumulation sector. Of course $H_Y + H_I = H$, where H is the total stock of human capital in the economy. In addition, it is assumed that F and G are both concave CRS production functions, while the function h is increasing. In this formulation, K usually stands for the stock of knowledge. The model has no externalities.

It is clear from (31) that if h(K)/K is bounded away from zero and $G(H_I) > 0$, then

$$\hat{K} = G(H_I)\, h(K)/K \tag{32}$$

will be positive in the limit as $K \to \infty$. So a positive and constant asymptotic growth rate of K is possible. But notice then how (30) implies that

$$\hat{Y} = \dot{Y} / Y = F_1(K, L, H_Y)\, \dot{K} / Y = [KF_1(K, L, H_Y)/ Y]\, \hat{K} \tag{33}$$

So when $\hat{K}$ is positive in the limit, a necessary and sufficient condition for a constant and positive rate of growth of output to be possible is that the ratio $KF_1(K, L, H_Y)/Y$, which represents the "share of knowledge" in the value of output, should be bounded away from zero in the limit as K goes to infinity.

Two important special cases of this model are now discussed.

V.1 *Expanding Input Variety*

Romer (1990) assumes that some firms undertake research with the deliberate intention of "discovering" new varieties of intermediate inputs. But an unintentional consequence of this research is that it becomes easier for other firms to innovate. His model can be expressed by means of the accumulation equation (31) together with the following three equations, whose meaning should be clear from the preceding analysis

$$Y = L_Y^{\beta} H_Y^{\delta-\beta} Z^{1-\delta}, \quad 0 < \beta < \delta < 1 \tag{34}$$

$$Z = \left[\int_0^{\infty} z(j)^{\alpha} dj\right]^{1/\alpha}, \quad 0 < \alpha < 1 \tag{35}$$

$$z(j) = \begin{cases} 0 & \text{if } K(j) < 1 \\ L(j) & \text{if } K(j) \geq 1 \end{cases} \tag{36}$$

Here L and H represent the levels of unskilled and skilled labor respectively. As before, we represent the set of varieties available on the market by the interval [0, n], where now n = K since K is only used to produce intermediate goods. Therefore, K can be interpreted as the measure of both the stock of knowledge and the variety of intermediate goods.

As in the previous section, the production system (34)–(36) exhibits love of variety for inputs. Also as in the derivation of (27) above, it can be seen that instantaneously efficient production here requires $z(j) = z = L_Z/K$ for all $j \leq K$. Hence $Z = [\int_0^n (L_Z/K)^{\alpha} dj\ ]^{1/\alpha} = [n\ (L_Z/K)^{\alpha}]^{1/\alpha} = K^{\phi} L_Z$, where ϕ is the constant $(1-\alpha)/\alpha$. So

$$Y = L_Y^{\beta} H_Y^{\delta-\beta} (K^{\phi} L_Z)^{1-\delta} \tag{37}$$

Now an efficient allocation of labor requires that $L_Y = \zeta L$, where $\zeta = \beta/(\beta+1-\delta)$. The system (34)–(36) can therefore be reduced to a version of (30) given by

$$Y = aK^{\phi(1-\delta)} L^{\beta+(1-\delta)} H_Y^{\delta-\beta} \tag{38}$$

where $a \equiv \zeta^{\beta}(1-\zeta)^{1-\delta}$. This must be supplemented by the accumulation equation (31), of course.

V.2 *Improving Input Quality*

Romer’s approach postulates innovation of new capital goods that make production of final goods less costly. In contrast, Grossman and Helpman (1991b, c), together with Aghion and Howitt (1992), have developed “quality ladder” models where innovation improves the quality of existing varieties of capital goods.[13] Since Grossman and Helpman assume that innovation

[13] In Aghion and Howitt, technical progress improves the quality of intermediate goods. Together with Segerstrom, Anant and Dinopoulos (1990) and Segerstrom (1991), they emphasize the links between endogenous innovation and Schumpeter’s process of “creative destruction.” Most of Grossman and Helpman’s work postulates upgrading the qualities of consumer goods. Such formulations are formally equivalent to ours because what matters in the end is the efficiency of capital goods in producing Gorman–Lancaster consumption characteristics. The capital goods formulation is more natural for our discussion, since we have been emphasizing intertemporal production possibilities throughout.

increases the quality of each good by some fixed proportion, in effect they obtain an equation for "knowledge accumulation" similar to (31). This can be seen in the following simple model that captures their main insights.

Output is assumed to be produced with a Cobb-Douglas production function

$$Y = D^{\beta} L_Y^{1-\beta}, \quad 0 < \beta < 1 \tag{39}$$

that uses labor L_Y and a composite input D. The latter is "assembled" from a continuum of varieties of inputs according to the function

$$D = \exp \int_0^1 \ln[\lambda^{m(j)} x(j)]\, dj = K \exp \int_0^1 \ln[x(j)]\, dj \tag{40}$$

where K is the stock of knowledge, which is given by

$$K = \exp \int_0^1 \ln[\lambda^{m(j)}]\, dj = \lambda \exp \int_0^1 m(j)\, dj \tag{41}$$

In these last two equations λ represents the proportional quality improvement that each innovation brings to input variety j, while m(j) measures the number of innovations that have happened to input variety j. Hence $\lambda^{m(j)}$ represents the quality of input variety j, measured as usual in efficiency units per physical unit.

Each input x(j) is produced from labor according to the simple production function x(j) = L(j), where x(j) and L(j) should be interpreted as output and input density functions on the interval [0, 1]. As before, (40) implies that it is instantaneously efficient to spread labor evenly across the different input varieties. Thus $x(j) = x = L(j) = L_X$ for all j in [0, 1]. Then (39) and (40) imply that

$$D = KL_X \text{ and } Y = (KL_X)^{\beta} L_Y^{1-\beta} \tag{42}$$

Instantaneous efficiency then requires that labor be distributed between sectors in constant proportions, with $L_X = \beta L$ and $L_Y = (1-\beta)L$. It follows that

$$Y = bK^{\beta}L, \text{ where } b \equiv \beta^{\beta}(1-\beta)^{1-\beta} \tag{43}$$

In the research sector, human capital is used to generate innovations to the different varieties of the input x. The innovation process is random, and described by a particular Poisson process. Putting H(j) units of human capital into research on variety j for a small time interval of length dt produces a quality upgrade with marginal probability aH(j)dt, where a is some positive constant.

This quality upgrade increases m(j) to m(j) + 1.[14] Because the different varieties of the input x enter (40) and (41) symmetrically, and because quality improvements are all proportional, it is instantaneously efficient to spread human capital evenly across varieties. Thus H(j) = H for all j. From (34) and the properties of the Poisson distribution (see Grossman and Helpman, 1991a, p. 97), it can then be seen that the stock of knowledge grows according to

$$\dot{K} = aHK \tag{44}$$

V.3 *Summary*

Recapitulating, we see that all three models (30)–(31), (31) with (34)–(36), and (43)–(44) basically amount to one common specification of technology. This requires final goods to be produced according to (30), where K represents the "stock of knowledge," which grows according to the equation $\dot{K} = G(H_I)\, h(K)$, as in (31) above.

VI. Endogenizing the Growth Rate

VI.1 *A More General Growth Model*

This section attempts to integrate into one single model the three types of growth models explored in sections III–V above. Moreover, section VII below will use the general analysis of this section in order to interpret earlier results on particular determinants of growth such as trade, finance, fiscal policy, social norms, etc.

The more general model we are about to present still uses very special functional forms, even though Hahn (1992) and others have rightly seen these as a serious limitation of most endogenous growth theory. However, what really matters for long-run growth is the asymptotic behaviour of the model as the economy becomes large. So it would actually be possible to reach similar conclusions after replacing the specific global functional forms we use with more general functions, while concentrating on their asymptotic elasticities and marginal products, etc. Such

[14]The more mathematically inclined reader may realize that there is a technical difficulty here, because a continuum of independent random variables produces non-measurable outcomes in a way that makes the integrals in (40) and (41) almost surely ill-defined. There are technical remedies, however, the easiest of which involves abandoning strict stochastic independence so as to make these formulae still applicable.

techniques are illustrated by the careful work of Brock and Gale (1969) and McFadden (1973) on the existence of optimal growth paths.

In our general model, but with specific functional forms, l_Y is the amount and u (= l_Y / l) is the proportion of labor which each agent devotes to production of final output (as opposed to education). Each agent's output y is assumed to depend on own physical capital k, own knowledge h, and efficiency units of labor $E\, l_Y$, where E is the common level of efficiency for all producers. As in section 4, labor has its efficiency enhanced by aggregate capital accumulation according to the equation

$$E = K^{\phi}, \quad \phi > 0 \tag{45}$$

Here it is important to recall that ϕ can be greater than one.

Each agent's concave production function is assumed to have a constant elasticity of substitution among factors that is greater one. Thus

$$y = F(k, h, E\, l_Y) = A[\beta k^{\alpha} + \delta h^{\alpha} + (1\text{-}\beta\text{-}\delta)(K^{\phi} u\, l)^{\alpha}]^{1/\alpha} \tag{46}$$

where $\alpha,\beta,\delta \in [0,1]$ with $0 < \alpha < 1$. Later, we shall also consider the Cobb–Douglas case when $\alpha \to 0$, and so $y = Ak^{\beta}h^{\delta}(K^{\phi}u\, l)^{1-\beta-\delta}$ with elasticity of substitution equal to one.

Output can either be consumed or invested to accumulate capital, so $y = c + \dot{k}$. Also, apart from being used to produce output, labor can also be allocated to each agent's own research sector, where it increases the stock of knowledge according to the equation

$$\dot{h} = \xi(l - l_Y)h^{\gamma} = \xi(1\text{-}u)\, l\, h^{\gamma}, \quad \xi > 0, \quad \gamma \in [0,1] \tag{47}$$

As in the particular models of section 5, a higher aggregate stock of knowledge makes research less costly. If one interprets h as the stock of human capital instead of knowledge, and $(1\text{-}u)\, l$ as the quantity of labor allocated to education, this formulation also captures the most important aspects of models in which skill acquisition is the engine of growth.

Compared to the neoclassical growth model, (46) and (47) incorporate all three departures that were considered in sections III to V. For example: if $\alpha > 0$, then none of the factors is essential for production; if $\phi > 0$, then the productivity of labor in the Y sector increases as a by-product of capital accumulation; while if $\gamma > 0$, then the productivity of labor in research increases as a by-product of previous research. If any of the three parameters α, ϕ, or γ were zero, however, the corresponding features would revert to those in the neoclassical model of Section II.

VI.2 *Endogenous Growth*

In order to determine the growth rate, this section also introduces, for the first time in this paper, an explicit intertemporal utility function. The continuum of identical and infinitely lived representative agents each maximize a utility integral over time by choosing independently how much income to save at all times, as well as what proportion u of labor time to devote to producing output. This kind of assumption has become standard in endogenous growth theory. It allows the equilibrium growth rate to be determined as a function of more fundamental demand side parameters, especially the discount rate and the degree of aversion to changes in consumption.

Let $\varepsilon \geq 0$ denote each representative agent's constant rate of relative fluctuation aversion (cf. the usual measure of relative risk aversion), and $\rho > 0$ the common instantaneous discount rate. Then each agent wants to maximize the intertemporal utility integral

$$U = \int_0^\infty e^{-\rho t} v(c)\, dt, \text{ where } v(c) = (1-\varepsilon)^{-1} c^{1-\varepsilon},\ \ \varepsilon > 0,\ \ \varepsilon \neq 1 \tag{48}$$

The special case $\varepsilon = 1$, in which the instantaneous utility function v(c) reduces to ln c, has received much attention in the recent literature.

Since E in (45) and (46) represents an externality, each agent does not take into account how this will be affected by private actions: instead, the time path K(t) of aggregate capital is taken as given. Therefore, each agent faces the same optimal control problem whose present value Hamiltonian $v(c) + r\dot{k} + q\dot{h}$ can be expressed as

$$(1-\varepsilon)^{-1} c^{1-\varepsilon} + r\{A[\beta k^\alpha + \delta h^\alpha + (1-\beta-\delta)(K^\phi u\, l)^\alpha]^{1/\alpha} - c\} + q\xi(1-u)\, l\, h^\gamma \tag{49}$$

Here r and q are the co-state variables of k and h respectively. Thus r is the shadow price of capital, while q is that of knowledge. As in Romer (1986) and Lucas (1988), an equilibrium can now be characterized by holding fixed the aggregate capital stock K in the usual conditions for this optimization problem, and then imposing the condition that K = k because all agents make the same choices in equilibrium. Necessary conditions for an equilibrium are that

$$c^{-\varepsilon} = r \tag{50}$$

$$rA^{\alpha}(1-\beta-\delta)(K^{\phi}\, l\,)^{\alpha}u^{\alpha-1}y^{1-\alpha} = q\xi\, l\, h^{\gamma} \tag{51}$$

and also

$$\dot{r} = \rho r - rA^{\alpha}\beta k^{\alpha-1}y^{1-\alpha} \tag{52}$$

$$\dot{q} = \rho q - rA^{\alpha}\delta h^{\alpha-1}y^{1-\alpha} - q\xi(1-u)\, l\, \gamma h^{\gamma-1} \tag{53}$$

where y is given by (46). Here, of course, (50) and (51) are the first order conditions for maximizing the Hamiltonian with respect to c and u respectively, while (52) and (53) are the co-state equations for K and h respectively.[15] Because (49) is concave in the current choice variables c, and u, equations (50) and (51) are sufficient for an appropriate global maximum at each instant of time. Sufficient conditions for intertemporal optimality are more subtle because the awkward last term $(1-u)\, l\, h^{\gamma}$ in (49) creates a non-concavity. However, it follows from the results of Seierstad and Sydsæter (1977) that the path derived *will* be optimal provided that: (a) the maximized Hamiltonian is concave in the two state variables k and h; (b) the standard transversality condition

$$e^{-\rho t}(rk + qh) \to 0 \text{ as } t \to \infty \tag{54}$$

is also satisfied. Unfortunately condition (a) in particular seems quite hard to verify in this model, except in special cases.

Recall the general notation $\hat{x}$ for the growth rate of any variable x. Then (50) implies that

$$\hat{c} = -\ \hat{r}\,/\,\varepsilon \tag{55}$$

Some manipulation of (52) together with (46) and the equilibrium condition K = k gives

$$\hat{r} = \rho - A\beta[\beta + \delta(h/k)^{\alpha} + (1-\beta-\delta)(k^{\phi-1}u\, l\,)^{\alpha}]^{(1-\alpha)/\alpha} \tag{56}$$

Equation (55) implies that for steady growth with $\hat{c}$ constant, the RHS of (56) must be constant. It is then obvious that there can never be balanced growth in this model, since no constant growth rate of h and k leaves this expression constant.

[15] It should be pointed out that really (51) is necessary only for an interior equilibrium, with $0 < u < 1$. But we shall be more concerned with (51) as one of a set of sufficient conditions.

VI.3 *First Case:* $\alpha > 0$ *and* $\gamma < 1$

In this case, the growth rate of h necessarily tends to zero because $\gamma < 1$. In the long run agents may as well choose $u = 1$. Then, if there is sustained growth of output, the economy necessarily approaches an asymptotic state in which c, y and k all grow at the same constant proportional rate g while h/k approaches zero. Moreover, as long as $-\rho + \hat{r} + g < 0$ so that the transversality condition $e^{-\rho t} rk \to 0$ as $t \to \infty$ is satisfied, the sufficient conditions for optimality described above will be met by any path satisfying (50)–(53): the awkward last term of (49) becomes zero.

In the subcase when $\phi < 1$ also, the term k^{ϕ} becomes relatively insignificant in the limit, and so we are essentially left with the "AK model" of section 3. Then (55) and (56) imply that as $k \to \infty$, so $\hat{r}$ tends to $\rho - A\beta^{1/\alpha}$ and $\hat{c} \to g$, where

$$g \equiv (A\beta^{1/\alpha} - \rho)/\varepsilon \tag{57}$$

The transversality condition requiring that $-\rho + \hat{r} + g < 0$ is also satisfied asymptotically in case $g < \rho - \hat{r} = A\beta^{1/\alpha}$. This requires that $A\beta^{1/\alpha} - \rho < \varepsilon A\beta^{1/\alpha}$ and so $\rho > (1-\varepsilon)A\beta^{1/\alpha}$. Hence, when ϕ and γ are both strictly less than one, sustained consumption growth at rate $g > 0$ occurs on a guaranteed optimal path if and only if $A\beta^{1/\alpha} > \rho > (1-\varepsilon)A\beta^{1/\alpha}$. Of course, when $\varepsilon \geq 1$ it is sufficient that $A\beta^{1/\alpha} > \rho > 0$, a condition similar in spirit to that derived by Jones and Manuelli (1990). This says that equilibrium consumption grows if and only if the marginal product of physical capital remains above the discount rate as the capital stock goes to infinity. On the other hand, if $0 < A\beta^{1/\alpha} \leq \rho$, then the rate of discount is so high that equilibrium consumption will not grow, even though such growth is feasible. But when $\rho \leq (1-\varepsilon)A\beta^{1/\alpha}$, the transversality condition (54) is violated, and the utility integral diverges on the asymptotic growth path. Then it is likely that no equilibrium exists because it is not optimal for any agent to follow the path whose asymptotic properties we have characterized given that all other agents follow the same path.

A knife-edge subcase occurs when $\gamma < 1$ and $\phi = 1$. The analysis is very similar to that in the previous paragraph, with the difference that the asymptotic growth rate becomes

$$g = (A^* - \rho)/\varepsilon, \text{ where } A^* \equiv A\beta[\beta + (1-\beta-\delta)\, l^{\,\alpha}]^{(1-\alpha)/\alpha} \tag{58}$$

Furthermore, the revised condition for existence of a growth equilibrium is $A^* > \rho > (1-\varepsilon)A^*$.

The last subcase to be considered is when $\gamma < 1$ and $\phi > 1$. Provided that $k \to \infty$ as $t \to \infty$, (55) and (56) imply that $\hat{r} \to -\infty$ and so $\hat{c} \to \infty$. Then (46) and the equilibrium condition $K = k$ together imply that $\hat{y} = \phi\,\hat{k}$. Therefore any equilibrium path must have all the growth rates $\hat{y}, \hat{k}$ and $\hat{c}$ converging to infinity as $t \to \infty$. Perpetual increasing returns lead to ever accelerating growth. As long as $\alpha > 0$ there is no (finite) asymptotic growth rate to determine in this case. Indeed, as Solow (1993) has observed in a very similar connection, there is even the possibility that output, consumption, and capital could all become infinitely large in a finite time.

VI.4 *Second Case:* $\alpha \to 0$

Since our aim is to determine steady state growth rates, we now restrict our analysis even further to the special case in which the production function is Cobb–Douglas. The equilibrium conditions (50)–(53) then need to be expressed in their limiting forms as $\alpha \to 0$, which we denote by (50')–(53'). Imposing the equilibrium condition $K = k$ on (52') gives

$$\hat{r} = \rho - \beta y/k = \rho - A\beta k^{-\psi} h^{\delta}(u\,l\,)^{1-\beta-\delta} \tag{59}$$

where $\psi \equiv 1 - (1-\beta-\delta)\phi - \beta = (1 - \beta)(1 - \phi) + \delta\phi$. Because of (55), steady growth of consumption at rate $g = \hat{c} = -\hat{r}/\varepsilon$ requires

$$\psi\hat{k} = \delta\hat{h} \tag{60}$$

To avoid rather perverse cases in which one of the variables h and k is required to shrink while the other expands, analysis will be restricted to the case when $\psi \geq 0$ because $\phi \leq (1-\beta)/(1-\beta-\delta)$.

The first subcase is when $\phi = (1-\beta)/(1-\beta-\delta)$ and so $\psi = 0$. Then a steady state requires h to be constant and so (47) implies that $u = 1$. When $\gamma \geq 1$ steady state growth is impossible, since it would not be an equilibrium to have $u = 1$. So we consider what happens when $\gamma < 1$.

Because $u = 1$ and $\psi = 0$, it follows from (46) and the condition $K = k$ that $y = Akh^{\delta}\,l^{\,1-\beta-\delta}$. So a steady state evidently requires growth at rate $g = \hat{c} = \hat{y} = \hat{k} = -\hat{r}/\varepsilon$. In addition, when $\alpha \to 0$, (51') becomes

$$r(1-\beta-\delta)y = q\xi u\,l\,h^{\gamma} \tag{61}$$

Taking the growth rates of all the variables in this equation when u = 1 and h is constant implies that $\hat{q} = \hat{r} + \hat{y} = g(1-\varepsilon)$. Since equation (53') gives $\hat{q} = \rho - r\delta y / qh$ when u = 1, it follows that

$$(1-\varepsilon)g = \rho - r\delta y / qh \tag{62}$$

Eliminating the ratio ry / qh from equations (61) and (62) then gives

$$(1\text{-}\varepsilon)g = \rho - \frac{\delta\xi l h^{\gamma-1}}{1-\beta-\delta} \tag{63}$$

when u = 1. Imposing the conditions $g = -\hat{r}/\varepsilon$, $\psi = 0$ and $u = 1$ on equation (59) then gives

$$-\varepsilon g = \rho - A\beta h^{\delta} l^{1-\beta-\delta} \tag{64}$$

Here (63) and (64) are two simultaneous equations in g and in h*, the asymptotic stationary level of h. By subtracting (64) from (63) it is easy to see that

$$g = A\beta h^{\delta} l^{1-\beta-\delta} - \frac{\delta\xi l h^{\gamma-1}}{1-\beta-\delta} \tag{65}$$

but in the general case there is no analytical solution for h*. In the special case when $\varepsilon = 1$, however, (63) implies that, for $\gamma < 1$, the stationary value of h is

$$h^* \equiv [\delta\xi l/\rho(1\text{-}\beta\text{-}\delta)]^{1/(1-\gamma)} \tag{66}$$

and then (64) determines the steady state growth rate as

$$g = A\beta(h^*)^{\delta} l^{1-\beta-\delta} - \rho \tag{67}$$

Finally, when $\varepsilon = 1$, it is easy to check that the transversality condition (54) is satisfied provided that $\rho > 0$.

The second subcase is when $\phi < (1\text{-}\beta)/(1\text{-}\beta\text{-}\delta)$ and so $\psi > 0$. Then (60) implies that $\hat{h}$ must be positive if the economy is growing. So $\gamma < 1$ would make long-run growth impossible. In case $\gamma = 1$, however, we look for balanced growth at a rate satisfying $g = \hat{c} = \hat{y} = \hat{k} = \delta\hat{h}/\psi = -\hat{r}/\varepsilon$ (where the last two equations follow from (47) and (60)), and with u constant. Then using (47), (59) and (60) yields

$$\hat{r} = -\varepsilon g = -\varepsilon(\delta/\psi)\,\xi(1\text{-}u)\,l, \text{ and so } g = (\delta/\psi)\,\xi(1\text{-}u)\,l \text{ while } \xi u l = \xi l - \psi g/\delta \tag{68}$$

Moreover, (53') together with (51') — or (61) with $\gamma = 1$ — implies that

$$\hat{q} = \rho - \xi(1\text{-}u)\,l - r\delta y/qh = \rho - \xi(1\text{-}u)\,l - \xi u l\delta/(1\text{-}\beta\text{-}\delta) \tag{69}$$

Next, differentiating (51') or (61) with $\gamma = 1$ and u constant gives $\hat{r} + \hat{y} = \hat{q} + \hat{h}$ and so

$$(1\text{-}\varepsilon)g = \hat{q} + \psi g/\delta = \hat{q} + \xi(1\text{-}u)\,l = \rho - \frac{\xi u l\delta}{1-\beta-\delta} = \rho - \frac{\xi l\delta - \psi g}{1-\beta-\delta} \tag{70}$$

Solving (70) for g gives the result that

$$g = \frac{(1-\beta-\delta)\rho - \xi l\delta}{(1-\varepsilon)(1-\beta-\delta) - \psi} = \frac{\xi l\delta - (1-\beta-\delta)\rho}{(1-\beta-\delta)(\varepsilon-\phi)+\delta} \tag{71}$$

where the second equality was derived using the definition of ψ. This solution is valid, and gives a positive growth rate, in case both numerator and denominator are positive, which they could be even if $\phi > 1$.

Finally, the transversality condition (54) is satisfied provided that $-\rho + \hat{r} + \hat{k}$ and $-\rho + \hat{q} + \hat{h}$ are both negative. It is routine to check that the second of these conditions is automatically satisfied because $u > 0$, while the first is also satisfied in case

$$\rho > \frac{(1-\varepsilon)\xi l \delta}{(1-\beta-\delta)(1-\phi)+\delta} \tag{72}$$

VI.4 *Summary of Results*

The most important clear cut results of this section are as follows:

Case (i) $\gamma < 1$, $\phi < 1$, $\alpha > 0$: In this case there is no equilibrium with long-run growth when $A\beta^{1/\alpha} \leq \rho$ or when $\rho \leq (1-\varepsilon)A\beta^{1/\alpha}$. But when $A\beta^{1/\alpha} > \rho > (1-\varepsilon)A\beta^{1/\alpha}$, the growth rate converges in the limit to $(A\beta^{1/\alpha} - \rho)/\varepsilon > 0$. This case corresponds to a model like those of Jones and Manuelli (1990) or Rebelo (1991) in which capital accumulation by itself generates long-run growth; that is, a neoclassical growth model without the upper Inada condition.

Case (ii) $\gamma < 1$, $\phi = (1-\beta)/(1-\beta-\delta)$, $\varepsilon = 1$ **and** $\alpha \to 0$: Here the steady state growth rate is

$$g = A\beta[\xi\delta l/\rho(1-\beta-\delta)]^{\delta/(1-\gamma)} l^{1-\beta-\delta} - \rho \tag{73}$$

provided that $\rho > 0$. In this case accumulation of h eventually stops, and the long-run engine of growth is a balance of capital accumulation with learning by doing; this is what makes ϕ equal to $(1-\beta)/(1-\beta-\delta)$. This case is formally equivalent to Romer (1986).

Case (iii) $\gamma = 1$, $\phi < (1-\beta)/(1-\beta-\delta)$ **and** $\alpha \to 0$: Here the steady state growth rate is:

g = **Error!**) (74)

provided that (72) is satisfied. In this case capital accumulation is only possible in the long run because h grows, and this is the true engine of growth.

In all three cases, when there is a steadily growing equilibrium, the long-run growth rate of consumption is decreasing in the discount rate, as is hardly surprising. Other parameters affect growth differently for the different cases. In cases (i) and (ii) capital accumulation is the engine of growth, and so it is intuitively reasonable that growth should be increasing in A. In cases (ii) and (iii), however, growth is increasing in both the size of the economy, as measured by each agent's stock of the fixed factor l, and the productivity of labor in education, as measured by ξ. This happens for different reasons in the two cases.

In case (ii), where capital accumulation combined with learning by doing is the engine of growth, a higher l affects growth by increasing the steady state value h^* of h, as indicated by (66). In turn, this occurs because a higher l increases the marginal product of h, and hence in equilibrium more resources will be devoted to increasing h. A similar argument explains why and how ξ affects the growth rate in case (ii).

On the other hand, in case (iii), human capital accumulation is the engine of growth. In this case either a higher l or a higher ξ affect the growth rate through a simpler mechanism: for each given 1-u, they both increase the growth rate of h. Finally, note that in case (iii) growth is increasing in the externalities from physical capital accumulation (ϕ) and, except possibly in rare cases when $\varepsilon < \phi$, is decreasing in the share $1-\beta-\delta$ of E in final output. The explanation for this last result is that a higher share of E in final output causes a reallocation of labor from this sector to the education sector, thus decreasing the rate of growth of h.

VII. Fundamental Determinants of the Rate of Growth

The previous section showed how the long run growth rate is determined by such parameters as the discount rate (ρ), the productivity of resources in production (A), the size of the economy (L), the intensity with which fixed resources are used in production ($1-\beta-\delta$), the productivity of the research sector (ξ), as well as the strength of externalities in production (ϕ) and in research (γ). Indeed, we have come a long way from the standard neoclassical growth model, where all these factors could have at most level effects.

The fundamental determinants of the rate of growth have not yet been properly considered, however, since it can be argued that many of these parameters are really determined endogenously by more fundamental variables like fiscal, trade and financial policy, the efficiency of the financial system, demographic variables and social norms. This section reviews some of the literature that has attempted to open up these black boxes. Wherever possible, these ideas will be linked to the general growth model of the previous section.

VII.1 *Trade and Growth*

Here the topic that has received most attention concerns how growth is affected by trade through its impact on the allocation of resources across sectors. Krugman (1987) and Lucas (1988) capture this in a simple model in which growth arises from learning by doing. They postulate a fixed quantity of labor L, which the economy allocates between the production of two goods z_1 and z_2. Producing one unit of good z_i requires q_i units of labor. Because of learning by doing in each sector, $dq_i/dt = -\nu_i L_i q_i$, where L_i is the quantity of labor allocated to the production of good i and ν_i is a positive parameter that measures the rate of learning by doing in sector i. Suppose that $\nu_1 > \nu_2$. When preferences exhibit a constant elasticity of substitution between the two goods, equilibrium in a closed economy involves a constant ratio L_i/L. So the growth rate of output is a weighted average of ν_1 and ν_2, with the weights given by L_i/L. If the economy is too small to affect international prices, then as it opens up to international trade, preferences in the economy no longer matter for the allocation of labor in equilibrium. Indeed, the economy will specialize in the good for which it has a static comparative advantage. If for some reason the comparative advantage of the economy is initially in good 2, the growth rate will decrease to ν_2. If the comparative advantage is in good 1 instead, the growth rate will increase to ν_1. In terms of the model developed in section 6, this is similar in spirit to having trade affect the learning-by-doing parameter ϕ. Young (1991) develops a more detailed and complete model along these lines.

In Grossman and Helpman (1991a, ch. 6), trade affects growth through a similar mechanism. Growth arises because new varieties of non-tradable intermediate goods are introduced, then used to produce two final goods. Innovation uses human capital intensively, and so does one of the final goods. If the economy has a static comparative advantage in the human capital-intensive good, then as the economy opens up to international trade, human capital will flow from the innovation sector to the production of this final good, thereby lowering the rate of growth. Of course, if the comparative advantage is in the good that uses human capital with low intensity, the growth rate will increase as the economy opens up to international trade. In terms of our model of section 6, trade affects growth by altering the share ($1-\beta-\delta$) of labor in the production of final goods.

A different effect of trade on growth is analysed by Stokey (1991). She considers a model in which growth arises from investment in education, which allows workers to produce more sophisticated (i.e., valuable) goods. Opening the economy up to international trade changes the relative prices of different goods, thus also changing the incentives for education. For instance, suppose the economy starts with a relatively low human capital stock. Then international trade will cause the prices of more sophisticated goods to drop relative to those of simpler goods for this economy, thus lowering both the incentives to become educated and the growth rate. The relation between this result and the analysis of section VI is harder to explain. However, the effect of trade on growth in this model is *like* the effect of a change in the share of human capital in the production of final goods (δ): that is, it affects the returns to human capital accumulation.

A different link between trade and growth is analysed by Grossman and Helpman (1991a, ch. 7 and 8) and by Romer and Rivera-Batiz (1991). The latter show that in a model similar to that in section 5.2 where innovation is the engine of growth, integrating two identical economies into one single market unambiguously increases the growth rate, since it increases the returns to innovation. This happens for the same reason that an increase in L increases the growth rate in case (iii) of section 6. If integration involves only trade, however, with no free flow of ideas, the authors show that it does not affect growth. But if innovation uses intermediate goods, then trade would increase the rate of growth because love of variety for inputs makes innovation less costly. This is analogous to a fall in ξ in the model of section 6. Grossman and Helpman also derive more general results along similar lines for the case in which the two economies are not identical.

VII.2 *Financial Intermediation and Growth*

A strong consensus among economists is that countries with healthier financial systems generally grow faster. Yet the model developed in section 6 has identical representative agents and must therefore lack individual uncertainty. This makes it impossible for financial intermediation to play any role in the growth process. Some recent papers nevertheless do analyse this phenomenon through growth models in which capital accumulation is the sole engine of growth. In terms of our growth model of section 6, all these models have the result that a more developed financial sector increases the value of the parameter A in (43). This is like output augmenting technical change, which leads to a higher growth rate as in cases (i) and (ii).

The financial system is given two main roles in these models. First, it encourages agents to invest a higher proportion of their wealth in more productive but riskier assets, as it reduces the need to keep wealth in liquid assets. This has been formalized recently by Bencivenga and Smith (1992), who allow agents to invest their wealth in two types of project: one with a high return and low liquidity; and another with a low return and high liquidity. In addition, agents face the possibility that in future they may need wealth in liquid form to meet an emergency. The authors show that, compared to a situation in which agents must finance their own projects, a better financial system allows more resources to be devoted to the high return investment, since it can meet the liquidity needs of agents without forcing them to invest in the low return projects.[16]

The other main role of financial intermediation is to allow agents with funds they do not need immediately to lend to other agents who have more pressing needs. It is crucial in modelling this role that there be lumpiness in project size and/or constraints on the maximum project size that an entrepreneur can manage. For otherwise investors could just invest all their own wealth without any need for borrowing and lending. Financial intermediation of this type introduces problems of asymmetric information, so that agents who borrow may not behave in a socially optimal way. For instance, as analysed by Aghion and Bolton (1991), agents who borrow will not devote as much effort to increasing the probability of success of the project. This makes wealth distribution matter for financial intermediation and growth. Note that this also has implications for the pattern of

[16] Another model along these lines is presented in Saint-Paul (1992). Also, Greenwood and Jovanovic (1990) develop a model in which the financial system increases the average productivity of investment and the growth rate, while growth reinforces the development of the financial system.

income inequality as the economy develops: the distribution of wealth at time t is a function of the distribution at time t-1. The authors discuss possible limits to this distribution.[17]

VII.3 *Demographics, Education and Growth*

The Uzawa–Lucas model of growth was reviewed in section 3, and is similar to case (iii) of section 6. It involves human capital being effectively transferred from generation to generation. Individuals spend the early part of their life acquiring skill, a process that is easier for individuals with well educated parents.[18] These models include a dynastic intertemporal utility function with discounting at the same rate over the lifetime of each (apparently single) parent as over that of each descendant.

Becker, Murphy and Tamura (1990) and Ehrlich and Lui (1992) extend these growth models to study the intergenerational process of growth in more detail. In Becker *et. al*., parents are altruistic and care about both the number of children they have and their education. Ehrlich and Lui assume that parents have children in order to receive care in their old age; the amount of care they get depends positively on the number of children and their education. In both these models better educated parents find it easier to educate their children. This leads them to opt for more education and fewer children. Therefore, as the economy grows and the average education increases, parents devote more time to education and less to bringing up many children. In turn, this reinforces the growth process. An interesting result derived by Ehrlich and Lui is that as the probability of children reaching maturity increases, parents invest more in education and so growth increases. On the other hand, when the probability of mature agents reaching old age increases, the effect on growth is ambiguous. In terms of our model in section 6, this work shows how the discount rate and hence the incentives to invest time in education depend on more fundamental demographic parameters.

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[17] Banerjee and Newman (1991) consider a related model. See Pagano (1992) and Stiglitz (1992) for more extensive analyses of the relationship between finance and growth.

[18] A similar but richer model appears in Stokey (1991), with the difference that a higher "social" level of human capital makes it easier for individuals to build their own human capital.

VII.4 *Fiscal Policy and Growth*

The effect of exogenous fiscal policy on the actual growth rate of an economy depends on how revenues are raised and on how the government spends those revenues. When the engine of growth is capital accumulation, and provided that the substitution effect dominates the income effect, income taxes that include taxation of interest income decrease incentives to accumulate capital, since owners can only obtain a fraction of the benefits — see Jones and Manuelli (1990) and Rebelo (1991). Such income taxes therefore have effects identical to those of lowering A in cases (i) and (ii) of the growth model of section 6. This differs drastically taxes on consumption expenditures and on investment in physical capital, which have no effect on the growth rate in the case where the engine of growth is human capital accumulation, as in case (iii) of section 6. For again the effect of a proportional tax on consumption and physical investment is identical to a decrease in A, which in case (iii) of section 6 has no effect on the growth rate.

On the other hand, as Barro (1990) argues, government expenditures in services that enhance productivity in the private sector will increase the growth rate. However, if revenues are used to finance government services that have no effect on productivity, or if they are wasted by bureaucrats, then growth of consumption will decrease. Of course, if the government services are desired by consumers, the implications for welfare remain ambiguous.

VII.5 *Social Norms, Politics and Income Inequality*

Many other organizations and institutions besides the market can affect growth. Perhaps most research has been devoted to understanding the effect of governments, whose powers may be used for income redistribution and abused by rent-seekers. In contrast to the previous subsection, the models we review here endogenize government policy decisions such as those affecting taxation.

Murphy, Shleifer and Vishny (1991) consider how the most talented people in society choose between becoming entrepeneurs, or rent-seekers whose activities reduce growth. It is shown that in making this choice, the most talented individuals in society will consider the size of the market in both activities, the size of the firm they can manage, and the type of contracts that can be enforced.

In general, the smaller the market for goods (say, because of poor infrastructure), or the smaller the size of the firm that can be effectively managed (say, because the capital market is underdeveloped), or the less that entrepreneurs can appropriate from the surplus they generate (say, because of unclear property rights and/or lack of patent protection), the more likely are the most talented individuals to become rent-seekers who lower the growth rate.

The government may also be used as an agency for income redistribution. Alesina and Rodrik (1991) and Persson and Tabellini (1991) both consider democracies in which fiscal policy that affects growth is decided by voting. The growth engine of Alesina and Rodrik's model is similar to Barro's (1990). In Persson and Tabellini's paper, however, growth is driven by the accumulation of knowledge in an economy where individuals have different abilities. Also, their fiscal policy involves only income redistribution. Both papers reproduce the traditional result that, when the distribution of income is skewed to the right, implying that the median of the distribution is smaller than the average, then there is more taxation or more income redistribution. The new feature is that this leads to a lower growth rate rather than just to a lower level of aggregate income.

Perotti (1992) explores a more complex link between income distribution and growth. Growth results from accumulating human capital, but in his model some individuals cannot afford the fixed cost of acquiring education. Nor are loans available to finance investment in education. When the economy is poor, fiscal policy that makes the distribution of income more unequal may increase growth, as a bigger proportion of the population can then afford education. However, when the economy is rich, increasing growth requires making the distribution of income more equal, since this decreases the proportion of the population who cannot afford education.

In all the models reviewed so far, growth arises because agents desire to increase future consumption either for themselves or for their descendants. However, in reality people care about things other than market goods, like being able to marry well. Usually such goods cannot be bought in the market, but are acquired "through status." Cole, Mailath and Postlewaite (1991) develop an interesting model in which people may save in part to acquire the status that will lead themselves or their children to enjoy more valuable non-market goods. The authors show that if social norms base status on wealth, growth is higher than if social norms make status independent of wealth. Of course, an alternative model based on Veblen's notion of conspicuous consumption is likely to have lower growth.

VIII. Conclusion

To evaluate the importance of the new endogenous growth theory, we must first recognize that there *is* long-run growth in the real world. For instance, according to Maddison (1982) the average growth rate of income per capita in the United States between 1890 and 1970 was 2.3 percent per year. Solow (1957) showed how only an eighth of this growth (up to the time when he wrote) could be explained by increases in the capital–labor ratio due to capital accumulation; the rest was the famous Solow residual.[19] Following Solow, this residual can be explained as the result of continual technology shocks, each of which increases the level of steady state income per capita. The observed growth rate is then be no more than the result of continous adjustment of the system to these exogenous shocks. It seems much more fruitful, however, to *explain* how these technological shocks arise: this is what endogenous growth theory has attempted to do. In a sense, the word "endogenous" is even redundant: for if growth is not endogenous, then really there is no growth *theory*, but only a sophisticated accounting system. Note that some earlier growth theories, such as Arrow's learning by doing model or Shell's (1967, 1974) model of publicly financed technological growth, are endogenous in this sense.[20]

This paper has concentrated on the *theory* of growth, to the virtual exclusion of its empirical side. So it would be rash to "summarize" what the new growth theory teaches us about real economies. However, there is one theoretical point general enough to deserve emphasis. This is that a common feature of many growth models is their reliance on positive externalities in one form or another. This is important, because it implies that growth rates will in general be lower than optimal and that there is a role for good government to play in encouraging appropriate growth. It

[19] This estimate has since come under repeated critical scrutiny. Some of the most recent re-estimates are presented and discussed in Boskin and Lau (1992).
[20] For another early model of endogenous technical progress, see Conlisk (1969).

also marks a clear, if minor, departure from the heavy use macroeconomists have been making of optimal growth models.

A final remark is in order. There is an important sense in which almost none of these recent results are really new: it was already well known that trade, finance, fiscal policy, income inequality and social norms could affect macroeconomic performance. Before the appearance of endogenous growth models, however, these variables would generally have only level effects, which would show up in growth rates only during the adjustment phase. Of course, as Atkinson (1969) and others pointed out, this adjustment phase could easily last for rather a long time, even several generations. Nevertheless, the main contribution of the new literature is that it explains how these same variables can also affect the long-run rate of growth. Although modest in fundamentally new insights, this is still an important contribution. For instance, since we are unable to measure welfare effects with sufficient precision, it is probably better to limit taxes to those that depart from the always unattainable first best further in their short run than in their long run effects. In particular, we should try to avoid destroying the incentives for agents to promote long-run growth. Furthermore, we should be emphasizing the long-run growth benefits of supply side policies such as freer international trade and market integration, while readily conceding the possible short-run damage to those who work in or own parts of uncompetitive industries, and the general desirability of arranging some short run compensation for those who do suffer short-run damage.

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