Introduction to Finite Elements in Engineering

FOURTH EDITION

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PEARSON

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To our parents

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Preface

The first edition of this book appeared over 20 years ago and the second and third editions followed subsequently. Translations of the book appeared in Spanish, Korean, Greek, and Chinese languages. We received positive feedback from professors who taught from the book and from students and practicing engineers who used the book. We also benefited from the feedback received from the students in our courses for the past 30 years. We have incorporated several suggestions in this edition. The underlying philosophy of the book is to provide a clear presentation of theory, aspects of problem modeling and implementation into computer programs. The pedagogy of earlier editions has been retained and enhanced in this edition.

WHAT'S NEW IN THIS EDITION

- Introduction of the superposition principle.
- Treatment of symmetry and antisymmetry.
- Additional examples and exercise problems.
- The patch test.
- Beams and Frames chapter moved to follow the Truss chapter.
- Revised Excel VB programs.
- Programs in JAVASCRIPT to run on web browsers such as IE, Firefox, Google Chrome, or Safari.
- Executable *graphics* programs together with the source codes.
- Additional examples and exercise problems.

New material has been introduced in several chapters. Worked examples and exercise problems have been added to supplement the learning process. Exercise problems stress both fundamental understanding and practical considerations. Problem modeling aspects have been added from early chapters. Principle of superposition is introduced in Chapter 1. Symmetry and antisymmetry considerations in two-dimensional problems are clearly presented. There are additional examples and exercise problems. The patch test is discussed and related problems have been added. The included programs have a common structure, which should enable the users to follow the development easily. Programs in JAVASCRIPT have been added. This will enable the users to solve finite element analysis problems using web browsers such as IE, Firefox, Safari, or Google Chrome. Excel VB programs have been revised. All programs have been thoroughly checked. The downloadable program set includes executable versions for programs involving graphics. The programs have been provided in Visual Basic, Microsoft Excel/Visual Basic, MATLAB, and JAVASCRIPT, together with those provided earlier in QBASIC, FORTRAN, and C. The Solutions Manual has been updated.

Chapter 1 gives a brief historical background and develops the fundamental concepts. Equations of equilibrium, stress–strain relations, strain–displacement relations, and the principles of potential energy are reviewed. The concept of Galerkin's method is introduced.

Properties of matrices and determinants are reviewed in Chapter 2. The Gaussian elimination method is presented, and its relationship to the solution of symmetric banded matrix equations and the skyline solution is discussed. Cholesky decomposition and conjugate gradient method are discussed.

Chapter 3 develops the key concepts of finite element formulation by considering one-dimensional problems. The steps include development of shape functions, derivation of element stiffness, formation of global stiffness, treatment of boundary conditions, solution of equations, and stress calculations. Both the potential energy approach and Galerkin's formulations are presented. Consideration of temperature effects is included.

Finite element formulation for plane and three-dimensional trusses is developed in Chapter 4. The assembly of global stiffness in banded and skyline forms is explained. Computer programs for both banded and skyline solutions are given.

Beams and application of Hermite shape functions are presented in Chapter 5. The chapter covers two-dimensional and three-dimensional frames.

Chapter 6 introduces the finite element formulation for two-dimensional plane stress and plane strain problems using constant strain triangle (CST) elements. Problem modeling and treatment of boundary conditions are presented in detail. Formulation for orthotropic materials is provided.

Chapter 7 treats the modeling aspects of axisymmetric solids subjected to axisymmetric loading. Formulation using triangular elements is presented. Several real-world problems are included in this chapter.

Chapter 8 introduces the concepts of isoparametric quadrilateral and higher-order elements and numerical integration using Gaussian quadrature. Formulation for axisymmetric quadrilateral element and implementation of conjugate gradient method for quadrilateral element are given.

Chapter 9 presents three-dimensional stress analysis. Tetrahedral and hexahedral elements are presented. The frontal method and its implementation aspects are discussed.

Scalar field problems are treated in detail in Chapter 10. While Galerkin as well as energy approaches have been used in every chapter with equal importance, only Galerkin's approach is used in this chapter. This approach directly applies to the given differential equation without the need of identifying an equivalent functional to minimize. Galerkin's formulation for steady-state heat transfer, torsion, potential flow, seepage flow, electric and magnetic fields, fluid flow in ducts, and acoustics are presented.

Chapter 11 introduces dynamic considerations. Element mass matrices are given. Techniques for evaluation of eigenvalues (natural frequencies) and eigenvectors (mode shapes) of the generalized eigenvalue problem are discussed. Methods of inverse iteration, Jacobi, tridiagonalization, and implicit shift approaches are presented.

Preprocessing and postprocessing concepts are developed in Chapter 12. Theory and implementation aspects of two-dimensional mesh generation, least-squares approach to obtain nodal stresses from element values for triangles and quadrilaterals, and contour plotting are presented.

At the undergraduate level some topics may be dropped or delivered in a different order without breaking the continuity of presentation. We encourage the introduction of the Chapter 12 programs at the end of Chapter 6. This helps the students to prepare the data in an efficient manner.

We thank Professor Hongbing Fang, Mechanical Engineering and Engineering Science, UNC Charlotte; Professor Kishore Pochiraju, Department of Mechanical Engineering, Stevens Institute of Technology, Hoboken, New Jersey; Professor Subramaniam Rajan, Ira A. Fulton School of Engineering, Arizona State University; Professor Chris H. Reidel, A. Leon Linton Department of Mechanical Engineering, Lawrence Technological University, Michigan; and Professor Nicholas J. Zabaras, Sibley School of Mechanical and Aerospace Engineering, Cornell University, who reviewed our third edition and gave many constructive suggestions that helped us improve the book.

Complete self-contained computer programs with source codes in Visual Basic, Excel-based Visual Basic, MATLAB, FORTRAN, JAVASCRIPT and C to accompany the text are available at www.pearsonhighered.com/chandrupatla.

Tirupathi Chandrupatla expresses his gratitude to J. Tinsley Oden, whose teaching and encouragement have been a source of inspiration to him throughout his career. He expresses his thanks to many students at Rowan University and Kettering University who took his courses. He expresses his thanks to his colleague Paris von Lockette who gave valuable feedback after teaching the course using the second and third editions.

Ashok D. Belegundu thanks his students at Penn State for their feedback on the course material and programs.

We thank Marcia Horton who provided us guidance with the present and previous editions of the book. We thank our editor Norrin Dias, Tacy Quinn, Debbie Yarnell, and Clare Romeo all from Prentice Hall who turned this project into a pleasant chore. We thank Maheswari PonSaravanan, project manager and her team at TexTech International, Chennai, India for efficient handling of copyediting and page proofs.

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About the Authors

Tirupathi R. Chandrupatla is Professor and Founding Chair of Mechanical Engineering at Rowan University, Glassboro, New Jersey. He received the B.S. degree from National Institute of Technology (formerly REC), Warangal. He received the M.S. degree in design and manufacturing from the Indian Institute of Technology (IIT), Bombay. He started his career as a design engineer with Hindustan Machine Tools, Bangalore. He then taught in the Department of Mechanical Engineering at IIT, Bombay. He pursued his graduate studies in the Department of Aerospace Engineering and Engineering Mechanics at the University of Texas at Austin and received his Ph.D. in 1977. He subsequently taught at the University of Kentucky. Prior to joining Rowan, he was a Professor of Mechanical Engineering and Manufacturing Systems Engineering at Kettering University (formerly GMI Engineering and Management Institute/General Motors Institute), where he taught for 16 years.

Dr. Chandrupatla has broad research interests, which include finite element analysis, design, optimization, quality and reliability, and manufacturing engineering. He has published widely in these areas and serves as a consultant to industry. Dr. Chandrupatla is a registered Professional Engineer and also a Certified Manufacturing Engineer. He is a member of ASEE, ASME, SAE, and SME.

Ashok D. Belegundu has been a Professor of Mechanical Engineering at The Pennsylvania State University, University Park, since 1986. Prior to this, he taught at GMI, now Kettering University, in Michigan. He received his B. Tech. degree from IIT, Madras and his Ph.D. from the University of Iowa. He has been a principal investigator on research projects involving finite element based optimization for several agencies, including the National Science Foundation, Army Research Office, NASA, SERC (UK), MacNeal-Schwendler Corporation, Gentex Corporation, and Ingersoll-Rand. A detailed list of his publications and projects can be found at http://www.mne.psu.edu/Directories/Faculty/Belegundu-A.html. He has given short courses on finite elements to the Forging Industry Association, Hazleton Pumps, Infosys (India).

Start by marking â€œIntroduction to Finite Elements in Engineeringâ€ as Want to Read: Want to Read savingâ€¦ Want to Read.Â This book provides an integrated approach to finite element methodologies. The development of finite element theory is combined with examples and exercises involving engineering applications. The steps used in the development of the theory are implemented in complete, self-contained computer programs. While the strategy and philosophy of the previous editions has been ret. This book provides an integrated approach to finite element methodologies. The development of finite element theory is combined with examples and exercises involving engineering applications. COÂ· ROM Included. Introduction to. Finite elements in engineering. Third edition. Tirupathi R. Chandrupatla Ashok D. Belegundu.Â body force in element e distributed to the nodes of the element traction fOfce in element e distributed to the nodes of the element virtual displacement variable: counterpart of the real displacement u(x, y, z) vector of vinual displacements of the nodes in an element; cOWlterpart of q shape functions in t1)(coordinates, material matrix, strain-displacement malril. respectively. 2.9 Introduction to Finite Elements. Course Home. Syllabus.Â The finite element method typically uses polynomial functions inside each element. Furthermore, the approximation is usually required to be continuous from element to element. The simplest element which permits continuous functions would be to assume linear variations of \(x\) inside each element. This type of element is called a linear element (not too surprisingly). Using linear finite elements, a sample solution might look like that shown in Figure 2.36. Figure 2.36: A linear element solution on a mesh with constant element size, \(\Delta x_ j = 0.2\). A linear function can be described by