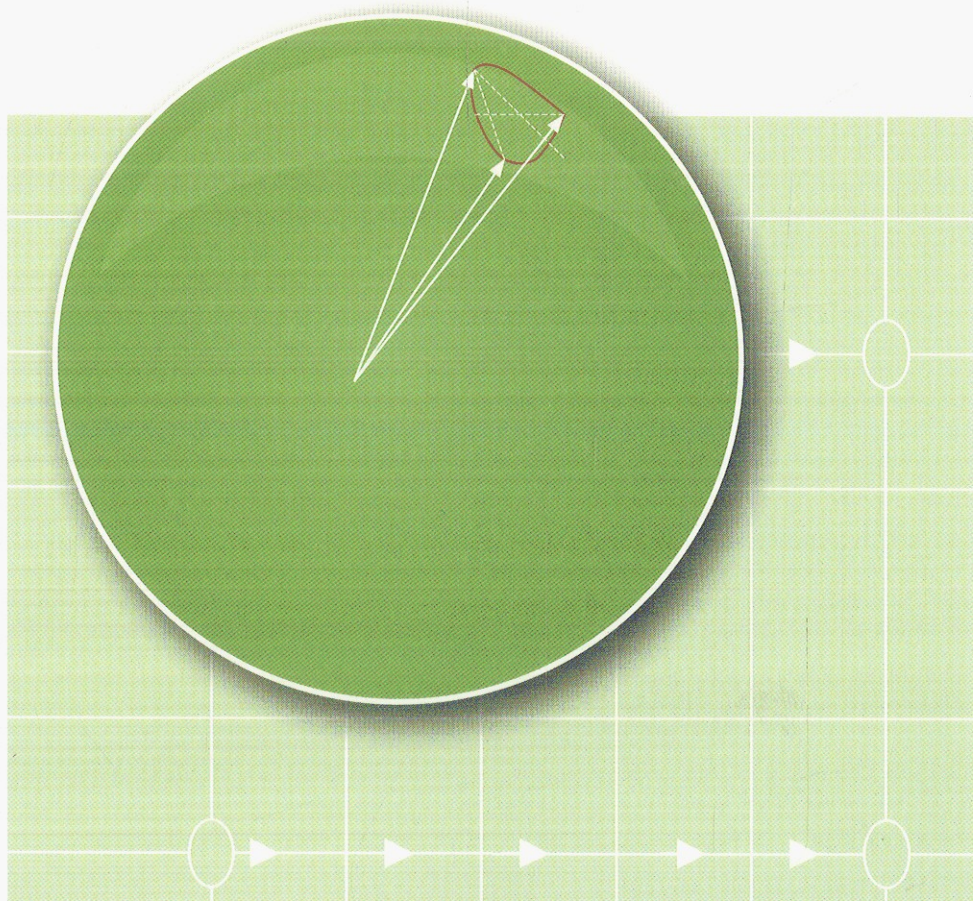


Tai-Kai Ng

 WILEY-VCH

Introduction to Classical and Quantum Field Theory



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Quantum Field Theory is a Relativistic Quantum Mechanics, but with fields instead of finitely many particle co-ordinates. The motivation comes from the principles of Locality and Lorentz-Invariance. Non-relativistic Quantum Field Theories exist, however, they are not Lorentz-Invariant. Quantum Field Theory shares many of the properties and equations, of Relativistic Quantum Mechanics; however, they are not applied to Wavefunctions, but Wave Functionals instead.